Test Review Questions Motion 4

Answers

Q1

Calculate the steady acceleration needed to take a cyclist from rest to $10\ \mathrm{m\,s^{-1}}$ in a distance of 20 m.

Use $v^2 = u^2 + 2as$

$100 = 0 + 2 \times a \times 20$

$\therefore\ a = 2.5\ \mathrm{m\,s^{-2}}$

Q2

A sprinter accelerates at 2.5 m s^{-2} for the first 4 s of a 100 m race, then keeps going at a steady pace.

(a) What is her top speed?

(b) What is her average speed over the 100 m race?

(a) Use $v = u + at$

$v = 0 + 2.5 \times 4$

$\therefore v = 10$ m s^{-1}

(b) Average speed = distance/time

Time taken for accelerating phase = 4 s

Distance travelled in accelerating phase:

$s = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 2.5 \times 16 = 20$ m

$\therefore$ Remaining 80 m were covered at 10 m s^{-1}

Time for constant speed phase = 8 s

Total time = 12 s

Average speed = 100/12 = 8.3 m s^{-1}

Q3

A stone is thrown up into the air. It reaches a height of 8 m. Assuming $g = 10\ m\,s^{-2}$, calculate its initial upward velocity and the time taken before it returns to its starting position.

Use $v^2 = u^2 + 2as$

Top of flight, $v = 0$, so

$0 = u^2 + 2 \times (-10) \times 8$

$u = \sqrt{160} = 13\ m\,s^{-1}$

Time taken to reach peak:

$v = u + at$

$t = (v - u)/a = (0 - 13)/(-10) = 1.3\ s$

Total time taken = 2.6 s

Q4

A car approaches a junction at 20 m s^{-1}. Traffic lights at the junction suddenly turn red when the car is 15 m away. The driver's reaction time is 0.5 s and the brakes decelerate the car steadily at 10 m s^{-2}. Where does the car stop?

Thinking distance $= 20 \times 0.5 = 10$ m

Braking distance: using $v^2 = u^2 + 2as$

$0 = 400 - 20 \times s$

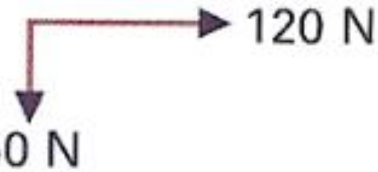
so $s = 20$ m

Total distance travelled by car $= 30$ m

It stops 15 m past the traffic lights!

Q5

The diagram below shows the two forces produced by a spoiler on a racin
car when it moves.



(a) Calculate the magnitude of the resultant force on the car.

(b) Calculate the direction the resultant force makes with the 120 N force.

(a) Magnitude: $R^2 = (120)^2 + (50)^2 = 14\,400 + 2\,500 = 16\,900$

$R = 130$ N

(b) Direction: $\tan\theta = 50 \div 120 = 0.4167$

$\theta = \tan^{-1}(0.4167) = 22.6°$

Q6

An aircraft carrier catapults a 12 000 kg jet from rest to 50 m s^{-1} in a distance of 50 m. Find the average force on the jet.

Use $\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$ to find acceleration.

$50^2 = 0 + 2 \times \boldsymbol{a} \times 50$ so $\boldsymbol{a} = 25$ m s^{-2}

$\boldsymbol{F} = 12\,000 \times 25 = 300\,000$ N

Q6

An object whose mass is 3.5 kilograms is traveling at 20. meters per second [east]. Calculate the momentum of the object.

$$\mathbf{p} = m\mathbf{v}$$
$$= (3.5\ \text{kg})(20.\ \text{m/s [E]})$$
$$= 70.\ \text{kg} \cdot \text{m/s [E]}$$

Q7

A bullet of mass 40 g is fired with a horizontal velocity of 500 m s^{-1} from a rifle of mass 2.5 kg.

Find: (i) the bullet's forward momentum, (ii) the bullet's kinetic energy, (iii) the speed of recoil of the rifle, (iv) the rifle's kinetic energy after the explosion.

(i) $\boldsymbol{p} = m\boldsymbol{v} = 0.04 \times 500 = 20$ kg m s^{-1}.
(ii) $E_k = \frac{1}{2}m\boldsymbol{v}^2 = 5\,000$ J. (iii) recoil speed = 20/2.5 = 8.0 m s^{-1}. (iv) rifle's $E_k = 80$ J.

Q8

What driving force is necessary to accelerate a car of mass 1 400 kg from rest to a speed of 35 m s^{-1} in 20 s?

$$a = 35/20 = 1.75 \text{ m s}^{-2}$$

$$F = 1\,400 \times 1.75 = 2\,450 \text{ N}$$

Q9

Two skaters are skating together at a steady velocity of 8 m s^{-1}. Their masses are 80 kg and 50 kg. The lighter skater is pushed forwards and accelerates to 10 m s^{-1}. Calculate the new speed of her partner.

Initial momentum of the two skaters is 130×8 $= 1040$ kg m s^{-1}. Final momentum of 50 kg skater is 500 kg m s^{-1}, so final momentum of 80 kg skater $= 540$ kg m s^{-1}. Final velocity $= 540/80 = 6.75$ m s^{-1}.

Q10

A 5.0-kilogram object traveling at 3.0 meters per second [east] is subjected to a force that increases its velocity to 7.0 meters per second [east]. Calculate: (a) the initial momentum of the object, (b) the final momentum of the object, (c) the change in momentum of the object, and (d) the impulse delivered to the object. (e) If the force acts for 0.20 second, what are its magnitude and its direction?

(a) $\mathbf{p}_i = m\mathbf{v}_i$
$= (5.0 \text{ kg})(3.0 \text{ m/s [E]})$
$= 15 \text{ kg} \cdot \text{m/s [E]}$

(b) $\mathbf{p}_f = m\mathbf{v}_f$
$= (5.0 \text{ kg})(7.0 \text{ m/s [E]})$
$= 35 \text{ kg} \cdot \text{m/s [E]}$

(c) $\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$
$= 35 \text{ kg} \cdot \text{m/s [E]} - 15 \text{ kg} \cdot \text{m/s [E]}$
$= 20. \text{ kg} \cdot \text{m/s [E]}$

(d) $\mathbf{J} = \mathbf{F}\,t = \Delta\mathbf{p}$
$= 20. \text{ N} \cdot \text{s [E]}$

(e) $\mathbf{J} = \mathbf{F}\,t$

$$\mathbf{F} = \frac{\mathbf{J}}{t}$$

$$= \frac{20. \text{ N} \cdot \text{s [E]}}{0.20 \text{ s}}$$

$= 100 \text{ N [E]}$

Q11

A 0.50-kilogram object traveling at 2.0 meters per second [east] collides with a 0.30-kilogram object traveling at 4.0 meters per second [west]. After the collision, the 0.30-kilogram object is traveling at 2.0 meters per second [east]. What are the magnitude and the direction of the velocity of the first object?

$$\mathbf{p}_{1_i} + \mathbf{p}_{2_i} = \mathbf{p}_{1_f} + \mathbf{p}_{2_f}$$

$$m_1\mathbf{v}_{1_i} + m_2\mathbf{v}_{2_i} = m_1\mathbf{v}_{1_f} + m_2\mathbf{v}_{2_f}$$

$$\mathbf{v}_{1_f} = \frac{m_1\mathbf{v}_{1_i} + m_2\mathbf{v}_{2_i} - m_2\mathbf{v}_{2_f}}{m_1}$$

$$= \frac{(0.5\text{ kg})(2\text{ m/s}) + (0.3\text{ kg})(-4\text{ m/s}) - (0.3\text{ kg})(2\text{ m/s})}{0.5\text{ kg}}$$

$$= -1.6\text{ m/s} = 1.6\text{ m/s [W]}$$

Q12

A 5.0-kilogram gun fires a 0.0020-kilogram bullet. If the bullet exits the gun at 800. meters per second [east], calculate the recoil velocity of the gun.

$$\mathbf{p}_{1_i} + \mathbf{p}_{2_i} = \mathbf{p}_{1_f} + \mathbf{p}_{2_f}$$

$$0 = \mathbf{p}_{1_f} + \mathbf{p}_{2_f}$$

$$0 = m_1\mathbf{v}_{1_f} + m_2\mathbf{v}_{2_f}$$

$$\mathbf{v}_{1_f} = \frac{0 - m_2\mathbf{v}_{2_f}}{m_1}$$

$$= \frac{0 - (0.0020\ \text{kg})(+800.\ \text{m/s})}{5.0\ \text{kg}}$$

$$= -0.32\ \text{m/s} = 0.32\ \text{m/s [W]}$$

Q13

How much work is done on an object if a force of 30 newtons [south] displaces the object 200 meters [south]?

$$
\begin{aligned}
W &= \mathbf{F} \cdot \mathbf{d} \\
&= (30\ \text{N}[\text{S}])(200\ \text{m}\ [\text{S}]) \\
&= 6000\ \text{J}
\end{aligned}
$$

Q14

As Alex pulls his red wagon down the sidewalk, the handle of the wagon makes an angle of 60° with the pavement. If Alex exerts a force of 100 newtons along the direction of the handle, how much work is done when the displacement of the wagon is 20 meters along the ground?

$$
\begin{aligned}
W &= (\mathbf{F} \cos\theta) \cdot \mathbf{d} \\
&= (100\ \text{N})(\cos 60°)(20\ \text{m}) \\
&= (50\ \text{N})(20\ \text{m}) \\
&= 1000\ \text{J}
\end{aligned}
$$

Q15

A car, accelerating uniformly at 2.0 m s^{-2}, has a speed of 20 m s^{-1} at time $t = 0$. Calculate the speed of and the distance travelled by the car at time $t = 10$ s. What is the speed of the car when it has travelled a distance of 400 m?

At time $t = 10$ s, speed of car,

$$v = u + at$$

$$= 20 + (2.0)(10) = 40 \text{ m s}^{-1} \quad \textbf{(ans)}$$

Distance travelled by car,

$$s = ut + \tfrac{1}{2} a t^2$$

$$= (20)(10) + \tfrac{1}{2}(2.0)(10)^2 = 300 \text{ m} \quad \textbf{(ans)}$$

At distance $s = 400$ m,

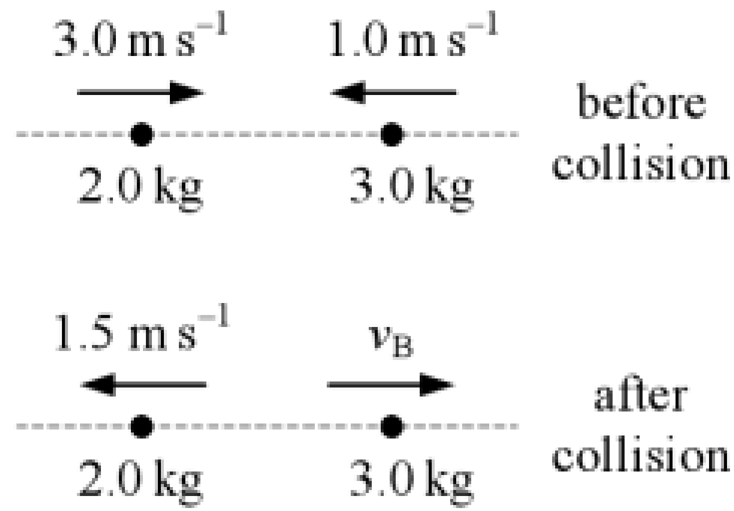
$$v^2 = u^2 + 2as$$

$$= (20)^2 + 2(2.0)(400) = 2000$$

$$v = \sqrt{2000} = 44.72 = 45 \text{ m s}^{-1} \text{ (2 sf)} \quad \textbf{(ans)}$$

Q16

Consider the head-on collision of two bodies A and B of masses 2.0 kg and 3.0 kg moving with velocities 3.0 m s^{-1} and – 1.0 m s^{-1} (or 1.0 m s^{-1} to the left) respectively before collision, and velocities – 1.5 m s^{-1} (or 1.5 m s^{-1} to the left) and v_B respectively after collision, as shown.



Conservation of momentum

total momentum before collision = total momentum after collision

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$ – in the usual notations

$(2.0)(3.0) + (3.0)(-1.0) = (2.0)(-1.5) + (3.0)\, v_B$

Velocity of B after collision, v_B = 2.0 m s^{-1} (to the right)

Q17

. An electric motor is used to raise a weight of 2.0 N through a vertical height of 80 cm in 4.0 s. If the efficiency of the motor is 20%, calculate the electric power supplied to the motor.

Useful power output of motor

$$= \frac{\text{work done lifting weight}}{\text{time taken}} = \frac{wh}{t}$$

$$= \frac{(2.0)(0.80)}{4} = 0.40 \text{ W}$$

$$\text{Efficiency} = \frac{\text{useful energy output}}{\text{energy input}} \equiv \frac{\text{useful power output}}{\text{power input}}$$

Electric power supplied to the motor

$$= \frac{\text{useful power output of motor}}{\text{efficiency of motor}} = \frac{0.40}{0.20} = 2.0 \text{ W (2 sf)}$$ **(*ans*)**

Q18

A car, accelerating uniformly at 2.0 m s^{-2}, has a speed of 20 m s^{-1} at time t = 0. Calculate the speed of and the distance travelled by the car at time t = 10 s. What is the speed of the car when it has travelled a distance of 400 m?

At time t = 10 s, speed of car, $v = u + at$

$= 20 + (2)(10) = 40$ m s^{-1} **(ans)**

Distance travelled by car, $s = ut + \frac{1}{2}at^2$

$= (20)(10) + \frac{1}{2}(2)(10)^2 = 300$ m **(ans)**

At distance s = 400 m, $v^2 = u^2 + 2as = (20)^2 + 2(2)(400) = 2000$

$\therefore v = \sqrt{2000} = 44.72 = 45$ m s^{-1} **(ans)**

Q19

At time $t = 0$ s, a ball is thrown vertically upwards on level ground and it reaches a maximum height of 50 m before falling back down. Assuming the value of g to be 9.81 ms^{-2}, calculate:

(a) The total time the ball is in the air.

(b) The initial speed at which the stone was thrown upwards.

(c) The speed of the stone when it is 25 m above the ground.

(d) The height of the stone at time $t = 5$ s.

Take velocity pointing downwards as positive.

(a) Total time ball is airborne = Twice the time ball takes to fall back to ground from its maximum height

Taking the time taken for ball to fall back to ground from maximum height as t:

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow 50 = 0 + \tfrac{1}{2} \times 9.81 \times t^2 \Rightarrow t = 3.192 \text{ s}$$

Total time stone is airborne = 2(3.192) = 6.38 s (3 sf) **(ans)**

Q19 continued

(b) The initial velocity of stone is taken as u.

$$v^2 = u^2 + 2as \Rightarrow 0 = u^2 - 2 \times 9.81 \times (50)$$

u = 31.3 ms^{-1} (3 sf) (ans)

(c) $v^2 = u^2 + 2as \Rightarrow v = \sqrt{31.3^2 + 2(-9.81)(25)} = 22.1\ ms^{-1}$ (3 sf) **(ans)**

(d) $s = ut + \frac{1}{2}at^2 \Rightarrow s = (-31.3) \times 5 + \frac{1}{2}(9.81)(5^2) = -33.9$

= 33.9 m above ground **(ans)**

Q20

A rocket propels itself upwards by discharging exhaust gases vertically downwards to produce an upward force on the rocket itself. When the engine of a certain rocket is started, it begins to eject exhaust gases at a rate of 30 kg per second at a constant velocity of 1000 $m\,s^{-1}$. Before ignition, the mass of the rocket and its fuel was 2000 kg.

(a) Calculate the initial weight of the rocket and its contents.

(b) There is an interval between ignition and take off. Explain why the time interval exists.

(c) Find the thrust and acceleration acting on the rocket when it takes off.

Q20 continued

(a) Initial weight = mg = 2000(9.81) = 19 620 N **(ans)**

(b) The rocket will only take off when thrust > mg, thus time is required for sufficient fuel to be burnt and increase its thrust. **(ans)**

(c) Thrust = $F = v\dfrac{\Delta m}{\Delta t}$ = $1000(30)$

= 30 000 N

Acceleration = $\dfrac{F}{m} = \dfrac{30000}{2000}$

= 15 ms^{-2} (3sf) **(ans)**