Motion Review

Physics 11 Week 10

Work, Energy & Power

- Work Done
- F causes an object to move a distance (been a transfer of E.
- Eg wheelbarrow – some E transf from person & becomes KE in the barrow

work done = force × distance (joules, newtons, metres) or, in symbols

$$E_w = F\,d$$

(J, N, m)

- Units are newton metres or joules
- Scalar quantity

Model Answer of Work Done

- Force of 20 N is used to push a box of 15kg 3.7m along a horizontal surface. Calculate the work done on the box.
- $E_w = Fd = 20 \times 3.7 = 74J$
- nb remember friction

Kinetic Energy Ek

- The E an object has when its moving
- The quantity depends on the objects m & v

$$\text{kinetic energy} = \frac{1}{2} \times \text{mass} \times \text{velocity}^2$$

(kinetic energy: joules; mass: kilograms; velocity: metres per second)

or, in symbols

$$E_k = \frac{1}{2} m v^2$$

(E_k: J; m: kg; v: m s^{-1})

- Units are joules
- The E depends on the square of the v
- To double cars v requires 4x's the E
- If mass x2 = x2 E to get same v

Model Answer of Ek

- Car of mass 600kg travelling at 80 m/s. Calculate its KE.

$$E_k = \frac{1}{2}\, m v^2 = \frac{1}{2} \times 600 \times 80^2 = 1.92 \times 10^6 \text{ J.}$$

- A 1 tonne car travelling at 30 m/s. How much energy will be needed to stop it?

$$E_K = \frac{1}{2} mv^2 = \frac{1}{2} 1000\ (30^2) = 500 \times 900 = 450\,000 \text{ J.}$$

- KE of car transf to heat by the brakes

Gravitational Potential Energy

- The E an object has because of its height.
- Quantity depends on m, g and vertical height

$$\text{potential energy} = \text{mass} \times g \times \text{height}$$

joules · *kilograms* · *N kg⁻¹* · *metres*

or, in symbols

$$E_p = m\,g\,h$$

J · *kg* · *N kg⁻¹* · *m*

- Units are joules

Model Answer of E_p

- A runner of mass 75kg at a height of 2.2m. Calculate PE
- E_p = mgh = 75 x 9.8 x 2.2 = 1617J
- A car (750kg) rolls down a frictionless slope (50m) with a vertical height of 10m. How much gravitational potential energy does it lose by the time it reaches the bottom?
- m x g x h = 750 x g x 10

PE to KE & Vice Versa

- Obj moves closer to centre = loss of PE
- As a result there is a gain in KE
- In absence of frictional forces the gain of KE = loss of PE

PE to KE & Vice Versa Formula

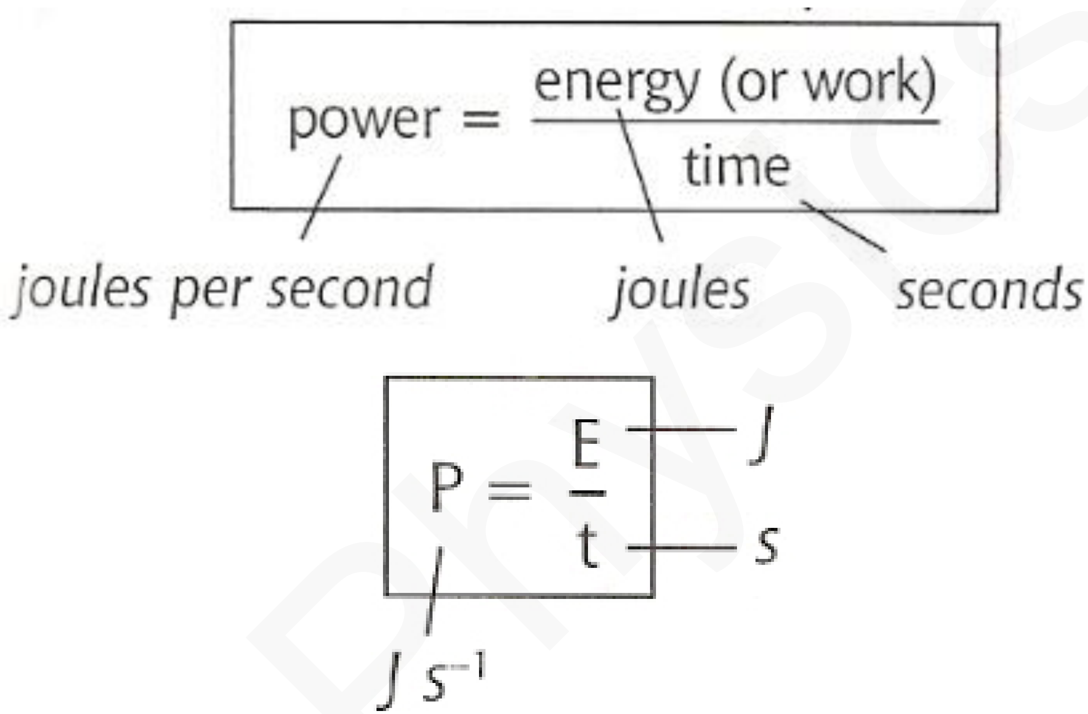
- Formula

$$\frac{1}{2} m v^2 = m g h \quad \text{Or} \quad v = \sqrt{(2 g h)}$$

- Can be used (abs of frictional forces) to find either
- The final v of obj dropped vertical h
- Find height reached by a obj launched with u

Power

- Rate of doing work (per s)



- Units are J (J/s) or watts (W)
- Scalar

Model Answer of P Question

- Lift can carry 12 people of average mass 85kg through a height of 20m in 15s. Calculate the minimum power output of lift’s motor.
- Total m lifted = 12 x 85 = 1020kg
- Work done = PE gained = mgh
- = mgh = (1020 x 9.8 x20) = 199 920J
- P = E/t = 199 920/15 = 13328 = 13kW

Power Example

- Car (800kg) travelling at 20m/s is brought to rest in 40s. What is the power of the brakes?
- Work done= KE lost
- $= \frac{1}{2} mv^2 = \frac{1}{2}$ x 800 x 20^2 = 160 000J
- Power =work done/time taken
- =160 000/40 = 4000W or 4kW

Example

A 20 kg block initially at rest has a force of 30 N applied to it for 5.0 seconds. Assume that the force of friction is constant and is 20 N. Find:

(a) The velocity after 5 seconds.
(b) The kinetic energy after 5 seconds.
(c) Work done by the 30 N force in this time.
(d) Why are (b) and (c) not equal?

(a)

F(nett)	=	10 N	F	=	ma
m	=	20 kg	∴ a	=	F/m = 10/20
u	=	0		=	$0.50\ ms^{-2}$
a	=	?	also v	=	u + at
t	=	5.0 s		=	0 + (0.5)(5)
v	=	?		=	$2.5\ ms^{-1}$

(b)
$$E_K = \tfrac{1}{2}mv^2 = (\tfrac{1}{2})(20)(2.5)^2 = 62.5\ J$$

(c)
$$s = ut + \tfrac{1}{2}at^2 = 0 + (\tfrac{1}{2})(0.50)(5)^2 = 6.25\ m$$

hence
$$W = Fs = (30)(6.25) = 187.5\ J$$

(d) Much of the work has been done to overcome friction and has been converted to heat (etc) rather than kinetic energy.

Example

A car of mass 1120 kg is travelling at 80.0 kmh^{-1} and slows down to 30.0 kmh^{-1} as it approaches a road works site. The car's reduced speed is achieved by applying the brakes over a distance of 100.0 m. Determine:

(a) The car's initial kinetic energy.
(b) The work done by the brakes in slowing the car down.
(c) The average force applied by the brakes.

(a)
$E_K = ?$
$m = 1120$ kg
$v = 80$ kmh^{-1}
$= 22.2$ ms^{-1}

$E_K = \frac{1}{2}mv^2$
$= (\frac{1}{2})(1120)(22.2)^2$
$= 2.76 \times 10^5$ J

(b)
$W = \Delta E_K$
$u = 80$ kmh^{-1}
$= 22.2$ ms^{-1}
$v = 30$ kmh^{-1}
$= 8.33$ ms^{-1}

Work done = Change in kinetic energy
$= E_{K\,(initial)} - E_{K\,(final)}$
$= (\frac{1}{2})(1120)(22.22)^2 - (\frac{1}{2})(1120)(8.33)^2$
$W = 2.76 \times 10^5 - 3.89 \times 10^4$
$= 2.38 \times 10^5$ J

(c)
$W = 2.37 \times 10^5$ J
$F = ?$
$s = 100$ m

$W = Fs$
$F = \frac{W}{s} = \frac{2.38 \times 10^5}{100}$
$= 2.38 \times 10^3$ N

Equations of Motion

- Relationships between s, v, a & t
- $v = u + at$
- $s = ut + \frac{1}{2} at^2$
- $v^2 = u^2 + 2as$
- $s = \frac{1}{2} (u + v)t$

s = displacement (m)

u = initial velocity ($m\ s^{-1}$)

v = final velocity ($m\ s^{-1}$)

a = acceleration ($m\ s^{-2}$)

t = time (s)

Choosing Appropriate Equations

- Make list of 3 quantities shown in question (some may be hidden – dropped obj – list u and a in this case)
- Add the quantities you have been asked for
- This list of 4 quantities indicates which equation to use

EoM Model 1 Answer

- Car travelling with v = 3 m/s. Accelerates at 2 m/s/s. Calculate the v after 8s ?
- u = 3, a = 2, t =8 v = ?
- Use v = u + at
- Substitution
- v = 3 + (2 x 8)
- = 19 m/s

EoM Model 2 Answer

- Brick dropped from tower block. How far will it fall after 3s?
- u = 0, a = 9.8, t = 3 s = ?
- $s = ut + \frac{1}{2}\ at^2$
- Substitution
- $s = (0\ x3) + (\ \frac{1}{2}\ x\ 9.8\ x\ 3^{2\,)}$
- $s = 0 + (\ \frac{1}{2}\ x\ 9.8\ x\ 9)$
- = 44.1 m

EoM Model 3 Answer

- Obj fired vertically upwards with an initial speed of 30 m/s. Calculate max height reached
- u = 30, v = 0, a = -9.8 s = ?
- $v^2 = u^2 + 2as$
- $0^2 = 30^2 + 2(-9.8)s$
- 0 = 900 -19.6s
- 19.6s = 900
- s = 45.92 m

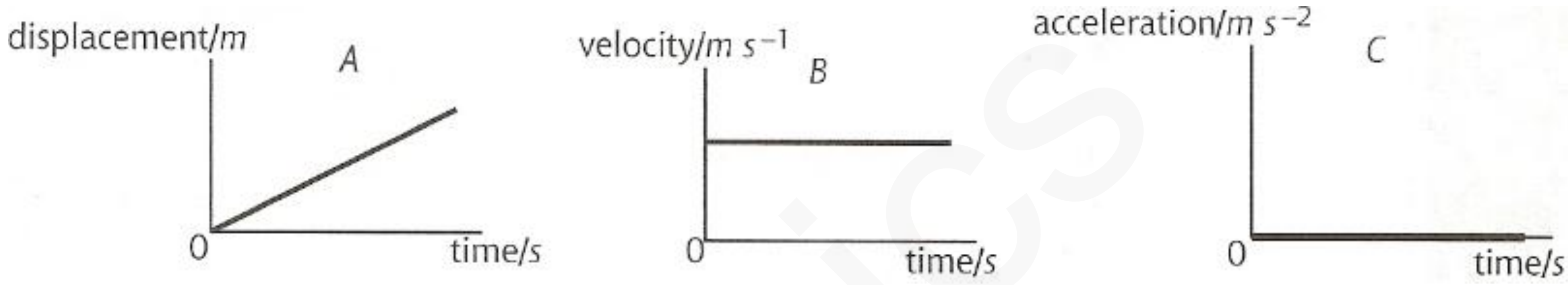
EoM Model 4 Answer

- Car accelerates uniformly from 5.6 m/s to 24.4 m/s in 8.5s. How far does it travel in this time?
- u = 5.6, v = 24.4, t = 8.5 s = ?
- $s = \frac{1}{2}(u + v)t$
- $s = \frac{1}{2}(5.6 + 24.4) \times 8.5$
- $s = \frac{1}{2}(30) \times 8.5$
- s = 127.5 m

Graphs of Motion

- 3 types
- d t graphs
- v t graphs
- a t graphs

Graphs for Constant Speed

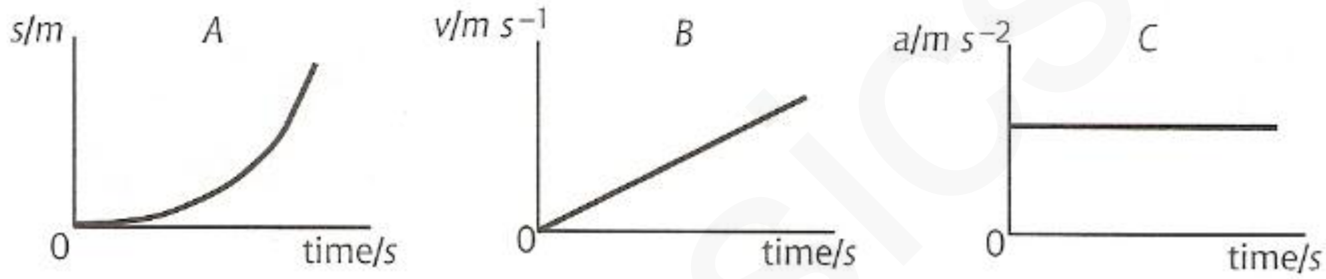


A – a straight diagonal line because a constant speed means equal distances are travelled in equal steps of time.

B – a horizontal line because the velocity is the same at all times.

C – a horizontal line at zero acceleration because the velocity is **not changing.**

Graphs for Constant Positive Acceleration

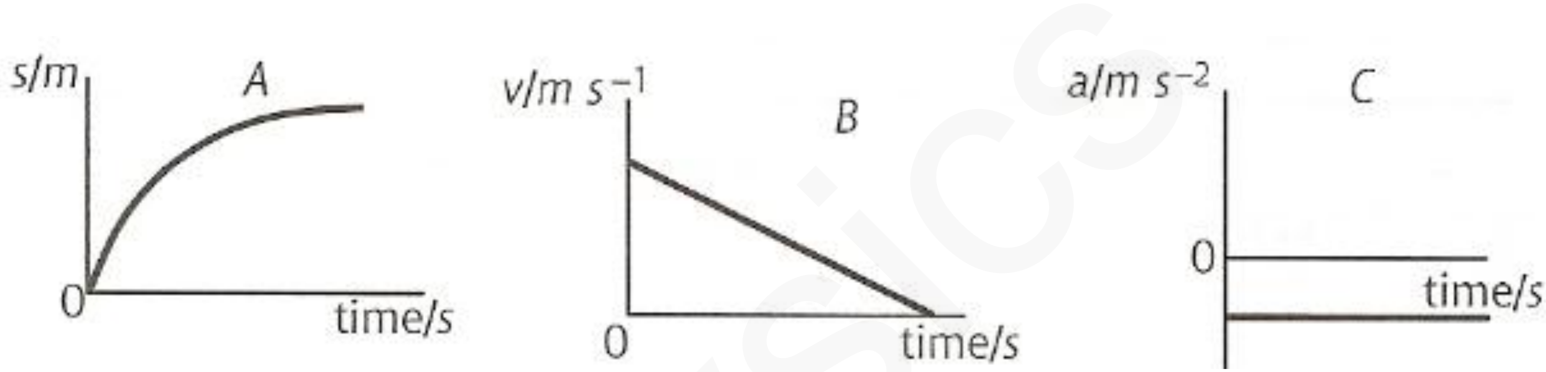


A – a line curving upwards because increasing speed means greater distances are travelled in equal steps of time.

B – a straight diagonal line sloping upwards because constant positive acceleration means velocity increases by equal amounts in equal steps of time.

C – a horizontal line because the acceleration is the same at all times.

Graphs for Constant Negative Acceleration



A – a line curving downwards because decreasing speed means smaller distances are travelled in equal steps of time.

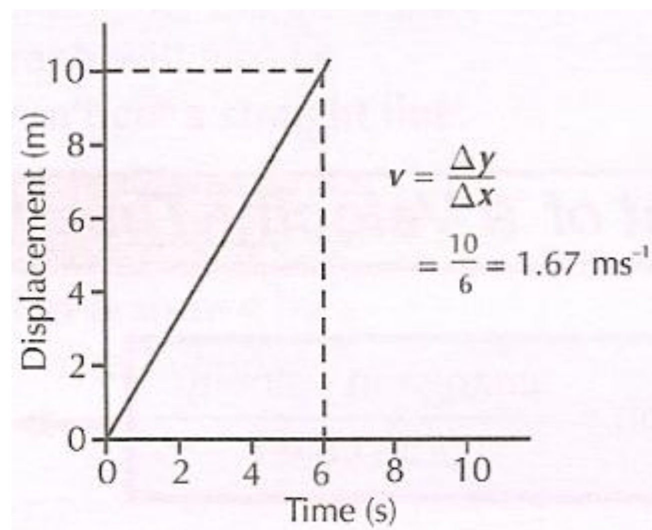
B – a straight diagonal line sloping downwards because constant negative acceleration means velocity decreases by equal amounts in equal steps of time.

C – a horizontal line because the acceleration is the same at all times. The line is below the time axis because the value of acceleration is negative.

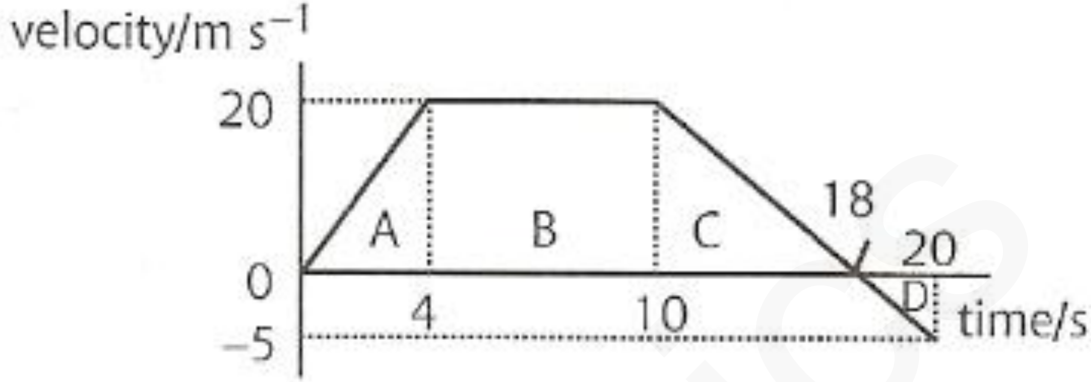
Graphs of Motion What They Tell Us

- Information can be calculated from GoM
- - the s is the area under a v/t graph
- - the a is the gradient of a v/t graph
- - gradient of a d/t graph tells you the v

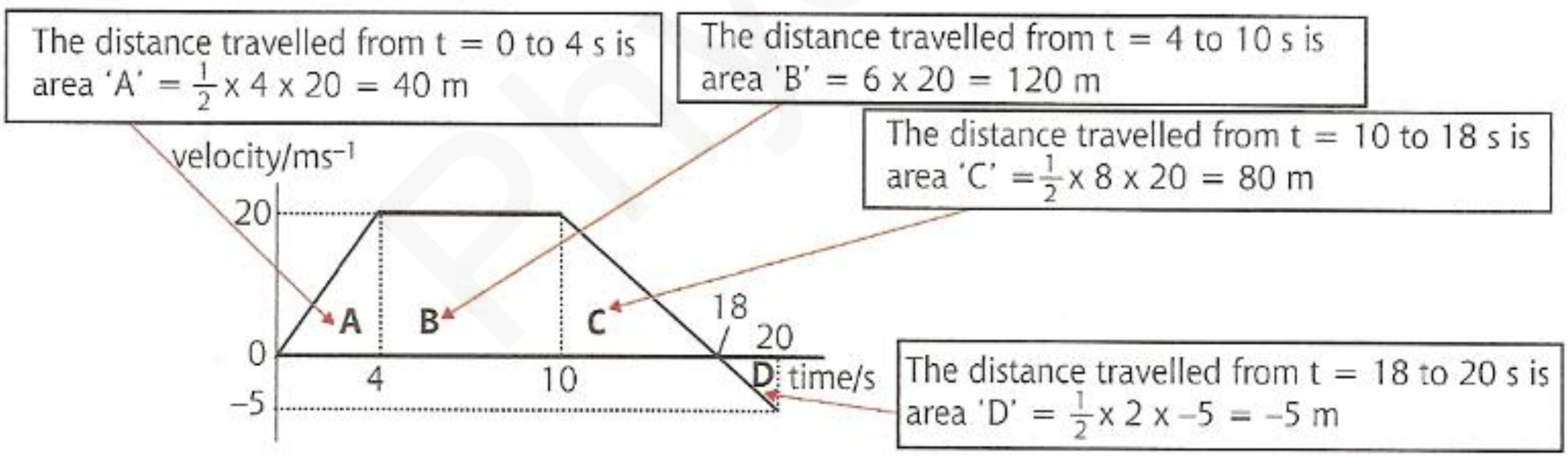
$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$



Example of What a v/t graph shows



- Find the displacement of the obj
- Draw the equivalent a/t graph



The total displacement = 40 + 120 + 80 − 5 = 235 m

Example of What a v/t graph shows continued

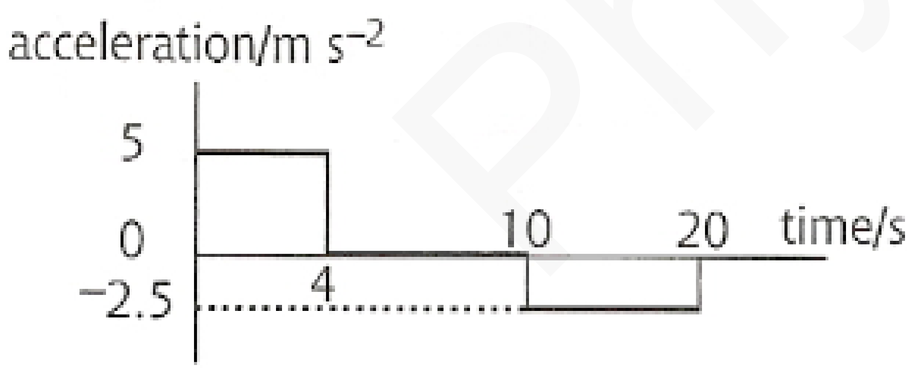
From t = 0 to 4 s, the acceleration = (v – u)/t = 20/4 = 5 m s^{-2}.

From t = 4 to 10 s, the acceleration = (v – u)/t = 0/6 = 0 m s^{-2}.

From t = 10 to 18 s, the acceleration = (v – u)/t = –20/8 = –2.5 m s^{-2}.

From t = 18 to 20 s, the acceleration = (v – u)/t = –5/2 = –2.5 m s^{-2}.

This gives the following graph.



Calculating Acceleration from a v/t Graph

- Can find a of an obj by calculating the gradient

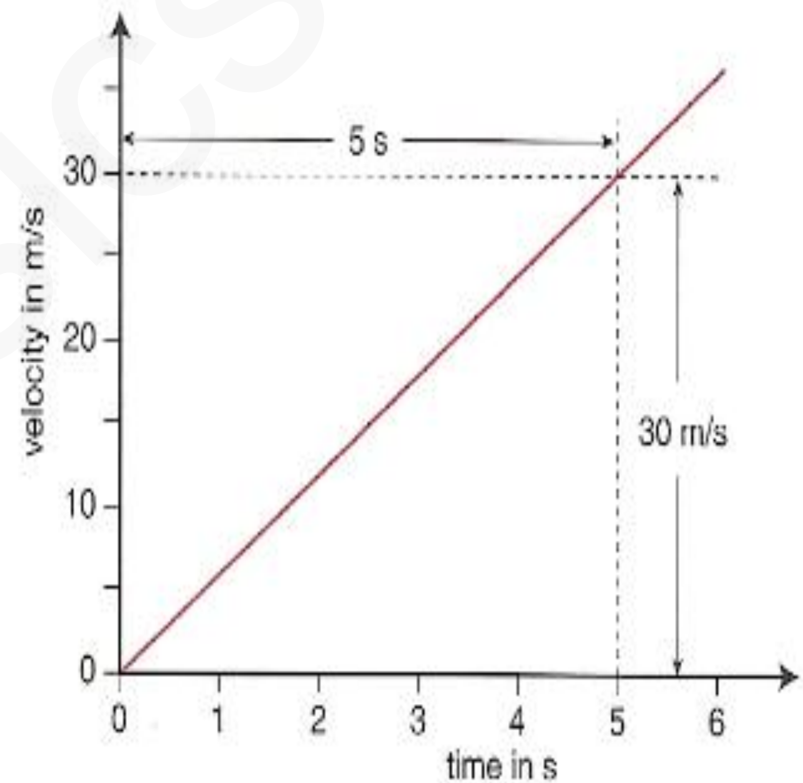
Calculating acceleration from a velocity–time graph

We can find the **acceleration** of an object by calculating the gradient of a velocity–time graph.

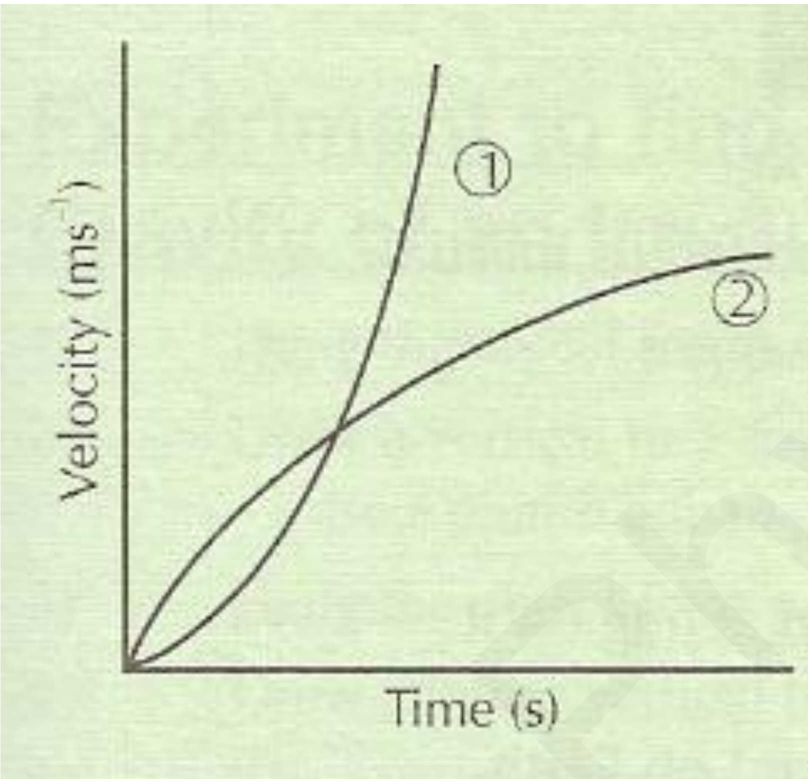
a = gradient of a v/t graph

The acceleration of the object in this graph is:

$$\frac{30\,\text{m/s}}{5\,\text{s}} = 6\,\text{m/s}^2.$$



Non-Uniform Acceleration on a v/t Graph



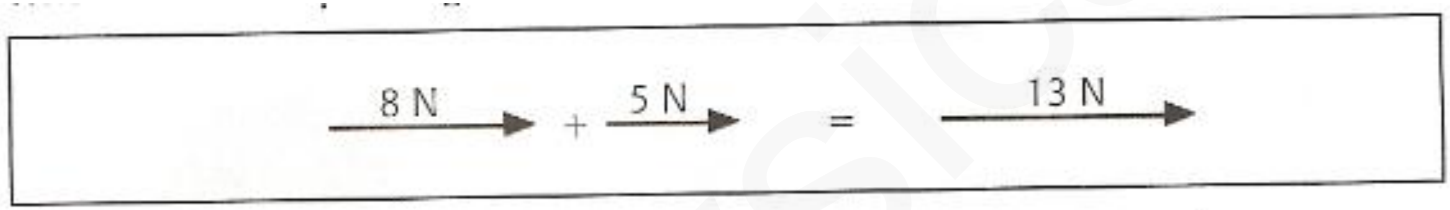
1) If the acceleration is changing, the gradient of the velocity-time graph will also be changing — so you **won't** get a **straight line**.

2) **Increasing acceleration** is shown by an **increasing gradient** — like in curve ①.

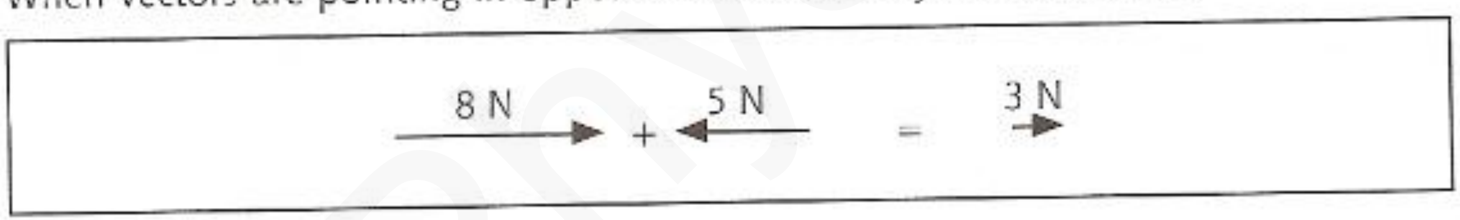
3) **Decreasing acceleration** is shown by a **decreasing gradient** — like in curve ②.

Vector Calculation

- Adding vectors along the same straight line



When vectors are pointing in opposite directions they are subtracted.



Adding Vectors in 2 Dimensions

- Normal arithmetic does not apply
- Resultant found by scale diagram or trig

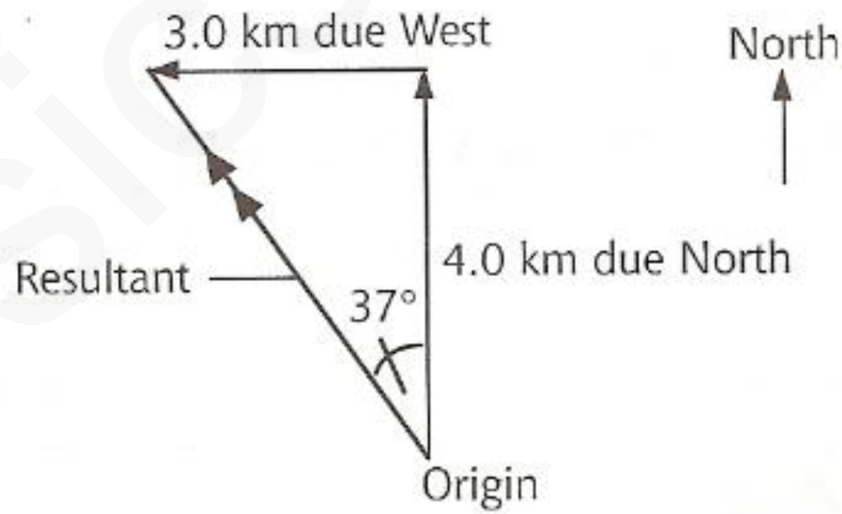
Combine Vectors by Scale Diagram

(a) Choose a suitable scale. (The diagram needs to be large enough or it will be inaccurate.)
(b) Choose a suitable point to start drawing the vectors – this is called the origin.
(c) Draw the first vector to scale and pointing in the correct direction.
(d) At the **end** of the first vector draw the second vector to scale and in its correct direction.
(e) Draw a straight line from the origin to the finishing point – this is the resultant.

Combine Vectors by Scale Diagram Model

A cross-country runner jogs 4.0 km due north and then turns and runs due west for a further 3.0 km. What is the runner's final displacement?

(a) A suitable scale could be 1 cm = 1.0 km.
(But note that a scale of 1 cm = 0.5 km would make a bigger diagram which would be more accurate.)



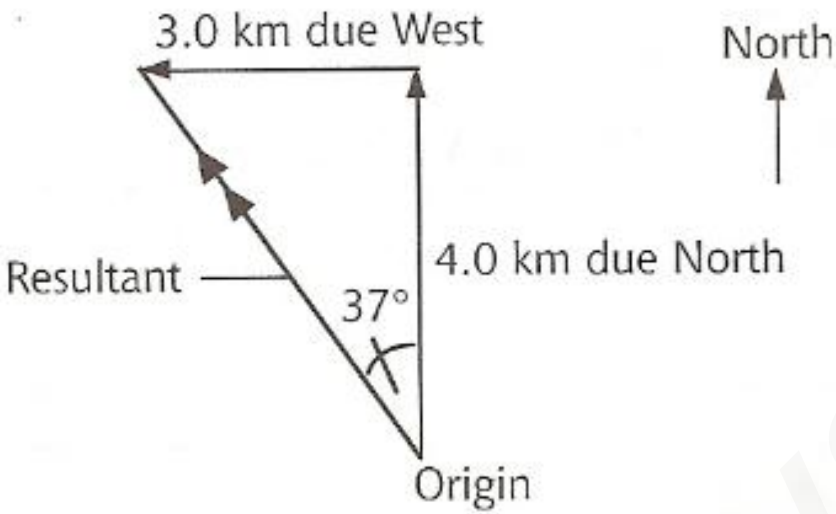
(b) The origin should be near the bottom of the page the right hand side – this allows space to draw th vectors upwards (north) and to the left (west).

Measurement on the diagram gives the length of the resultant vector to be 5.0 cm which is equivalent to 5.0 km in real life.

A protractor should be used to measure the shown angle.
This angle is 37° west of north, which is equivalent to a three-figure bearing of 323°.

Finally, the resultant should be quoted as '5.0 km bearing 323°'.

Combine Vectors by Geometry & Trigonometry Model



West and north are at right angles to each other. This means the triangle is right-angled and Pythagoras' theorem can be used to find the hypotenuse, i.e.

Resultant $= \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5.0$ km.

Tan $\times$ = opposite/adjacent $= \frac{3}{4} \Rightarrow \times = \tan^{-1} 0.75 = 37°$.

Combine Vectors by Geometry & Trigonometry Model

A person walks 5.0 km due south in 2.0 hours.
He then turns and walks a further 10.0 km on a bearing of 070°, taking a further 3.0 hours.

Calculate:

(a) the total distance travelled;

(b) his average walking speed;

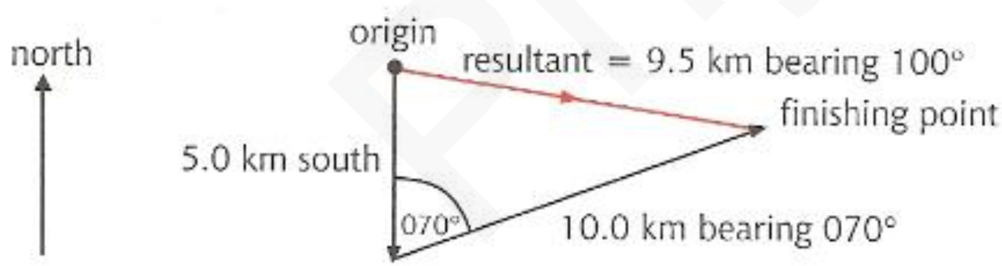
(c) his resultant displacement;

(d) his average velocity.

(a) Total distance = 5.0 + 10.0 = 15.0 km.

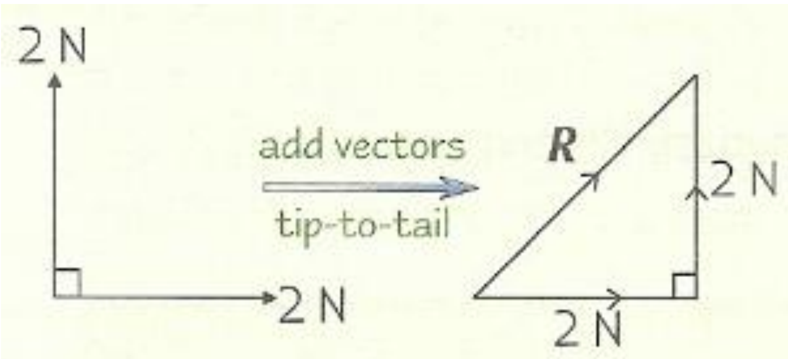
(b) Average speed = total distance/time taken = 15.0/5 = 3.0 km per hour.

(c) Scale 1.0 cm = 1.0 km.



(d) Average velocity = total displacement/time = 9.5/5 = 1.9 km h^{-1} bearing 100°.

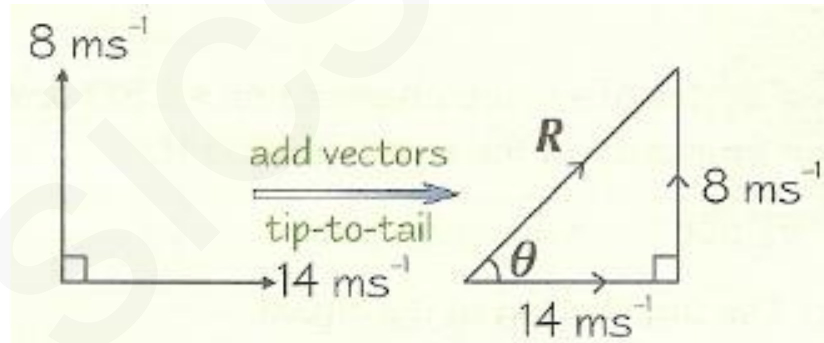
Other Model Answers



$R^2 = 2^2 + 2^2 = 8$

which gives $\boldsymbol{R} = 2.83$ N at 45° to the horizontal.

Remember SOH CAH TOA.



Start with: $\boldsymbol{R}^2 = 14^2 + 8^2 = 260$

so you get: $\boldsymbol{R} = 16.1\ \text{ms}^{-1}$.

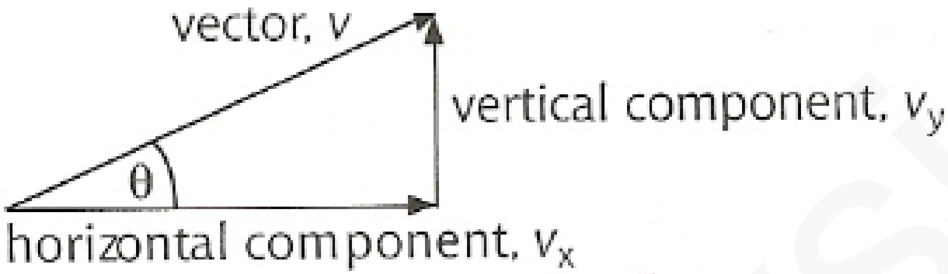
Then: $\tan\theta = 8/14 = 0.5714$

$\theta = 29.7°$

Resolving Vectors into Components

- Resolving = breaking it down into 2 vectors
- Usually at right angles
- Vector v at angle 0o to the horizontal can be resolved into its horizontal component & its vertical component

Resolving Vectors into Components Using Trig



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{v_x}{v}$$

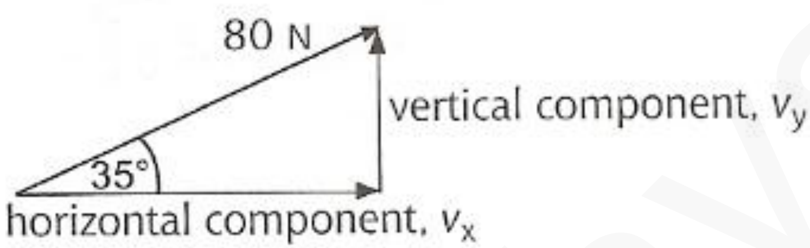
⇒ the horizontal component of the vector, $v_x = v\cos\theta$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{v_y}{v}$$

⇒ the vertical component of the vector, $v_y = v\sin\theta$

Model Answer For Resolving Vectors into Components Using Trig

A force of 80 N acts at an angle of 35° up from the horizontal. What are the horizontal and vertical components of this force?



the horizontal component of the vector, $v_x = v \cos \theta$

$= 80 \cos 35°$

$= 65.5$ N

the vertical component of the vector, $v_y = v \sin \theta$

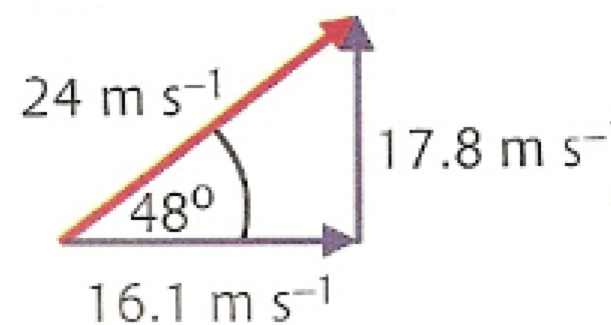
$= 80 \sin 35°$

$= 45.9$ N

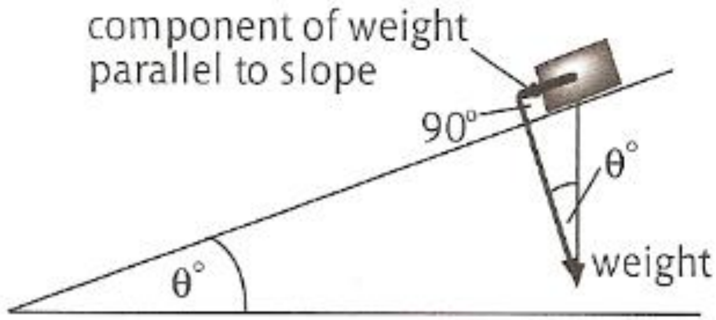
Model Answer For Resolving Vectors into Components Using Trig

An athlete throws a javelin with a speed of 24 m s^{-1} at an angle of 48° to the horizontal.
Calculate the horizontal and vertical components of its initial velocity.

Horizontal component of the initial velocity = $v \cos \theta$ = $24 \times \cos 48 = 16.1$ m s^{-1}.
Vertical component of the initial velocity = $v \sin \theta = 24 \times \sin 48 = 17.8$ m s^{-1}.



On a Slope



From the right-angled triangle:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{\text{component of weight down slope}}{\text{weight } (= mg)}$$

- Produces the following relationship

$$\text{Component of weight down slope} = m\,g\sin\theta$$

Model Answer of Object on a Slope

A cyclist and bicycle have a total mass of 110 kg.

The cyclist pedals uphill on a slope which makes an angle of 8° to the horizontal.

The cyclist exerts a driving force of 260 N. The force of friction is 40 N.

(a) Calculate the component of weight parallel to the slope.

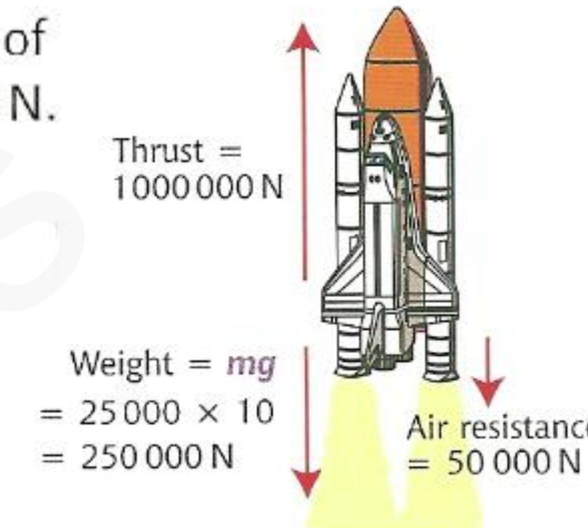
(b) Calculate the unbalanced force.

(a) Component of weight down slope = $m\,g\,\sin\theta = 110 \times 9.8 \times \sin 8° = 150$ N.

(b) Unbalanced force = driving force – (compt. of weight + friction) = $260 - (150 + 40) = 70$ N.

Vectors & Resultant Forces

A rocket of mass 25 000 kg is accelerating with a thrust of 1 000 000 N while experiencing air resistance of 50 000 N. Find the resultant force on the rocket and hence its acceleration.

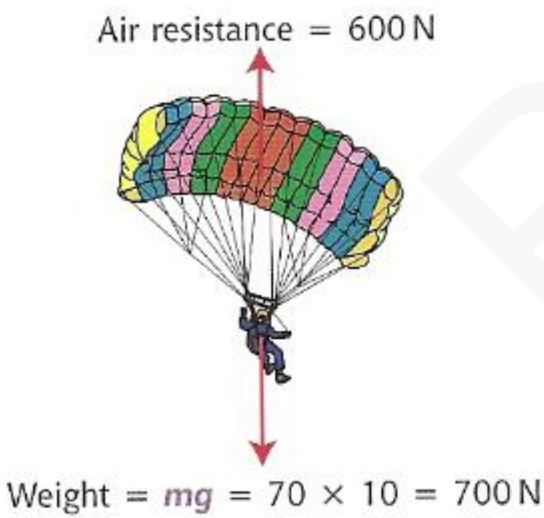


Resultant force

= total upward force – total downwards force

= 1 000 000 – 300 000

= 700 000 N upwards



A parachutist is descending towards the ground. At a certain point in time she experiences air resistance of 600 N upwards. If her mass is 70 kg determine her motion at that point.

Resultant force

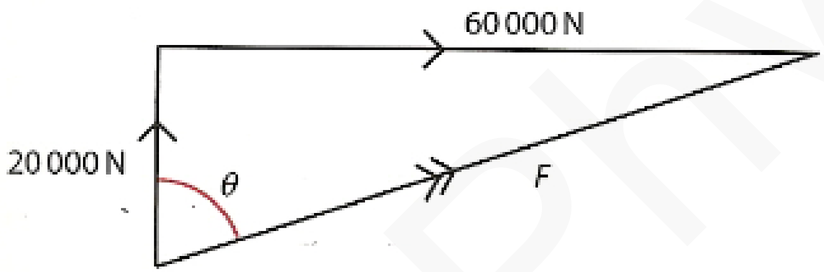
= total downwards force – total upwards force

= 700 – 600 = 100 N downwards

acceleration, $a = \frac{F_{un}}{m} = \frac{100}{70} = 1.4\ \text{m/s}^2$ downwards.

Vectors & Resultant Forces continued

Two tugs pull a ship off a pier. One pulls forward with a force of 60 000 N while the other pulls sideways with a force of 20 000 N. Calculate the resultant force on the ship.



$F = \sqrt{(20\,000^2 + 60\,000^2)} = 63\,245\ \text{N}$

$\tan\theta = \text{opp} / \text{adj} = 60\,000/20\,000 = 3$

$\theta = 71.6°$