

Momentum concepts

Physics Year 11 Term 1 Week 5

Momentum, p

momentum = mass x velocity

$$p = mv$$

m in kilograms (kg)

v in metres per second (ms^{-1})

p in kilograms metres per second (kg ms^{-1})

Momentum is a VECTOR quantity

– direction the same as the velocity

Force and Momentum

Force is equal to the rate of change of momentum.

$$F = \Delta(mv) / \Delta t$$

F in newtons (N)

$\Delta(mv)$ in kilograms metres per second (kg ms^{-1})

Δt in seconds (s)

Question 1

A car of mass 800 kg moving at a velocity of 30 ms^{-1} is brought to rest by a braking force of 1200 N.

Calculate:

(a) its initial momentum

(b) the time taken to stop the car.

$$(a) p = mv$$

$$= 800 \text{ kg} \times 30 \text{ ms}^{-1}$$

$$\text{momentum} = 24\,000 \text{ kg ms}^{-1}$$

$$(b) F = \Delta(mv) / \Delta t$$

$$1200\text{N} = 24\,000 \text{ kg ms}^{-1} / \Delta t$$

$$\Delta t = 24\,000 \text{ kg ms}^{-1} / 1200\text{N}$$

$$\text{time} = 20 \text{ seconds}$$

Question 2

A car of mass 750kg travelling at a speed of 4.0ms^{-1} is struck from behind by another vehicle. The impact lasts for 0.30s and causes the speed of the car to increase to 6.0ms^{-1} .

Calculate:

(a) the change in momentum of the car due to the impact.

(b) the impact force.

$$(a) \Delta p = \Delta mv$$

mass is constant, so:

$$\Delta p = m \Delta v$$

$$= 750 \text{ kg} \times (6.0 - 4.0) \text{ ms}^{-1}$$

momentum change

$$= 1\,500 \text{ kg ms}^{-1}$$

$$(b) F = \Delta(mv) / \Delta t$$

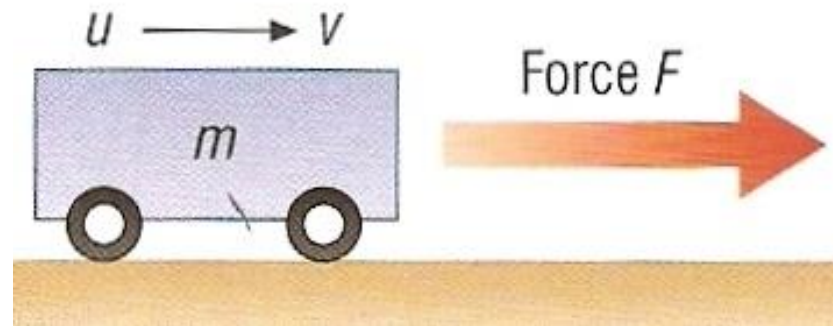
$$= 1\,500 \text{ kg ms}^{-1} / 0.30\text{s}$$

force = 5000 N

Answers

momentum	mass	velocity
150 kg m/s	50 kg	3 m/s
160 kg m/s	8 kg	20 m/s
1500 kg m/s	250 kg	6 m/s
4 kg m/s	500 g	8 m/s
3 kg m/s	6 kg	50 cm/s

Force and momentum



A force will cause the velocity of an object to change and therefore also its momentum.

The greater the force the faster the momentum changes.

$$\text{force} = \frac{\text{momentum change}}{\text{time taken for the change}}$$

force is measured in **newtons (N)**

change in momentum is measured in:

kilogram metres per second (kg m/s)

time is measured in **seconds (s)**

Question 1

Calculate the force required to change the momentum of a car by 24000 kgm/s over a 6 second period.

force = momentum change ÷ time taken

= 24000 kgm/s ÷ 6 s

force = 4000N

Question 2

Calculate the time taken for a force of 6000N to cause the momentum of truck to change by 42000 kgm/s.

force = momentum change ÷ time taken

becomes:

time taken = momentum change ÷ force

= 42000 kgm/s ÷ 6000 N

force = 7 seconds

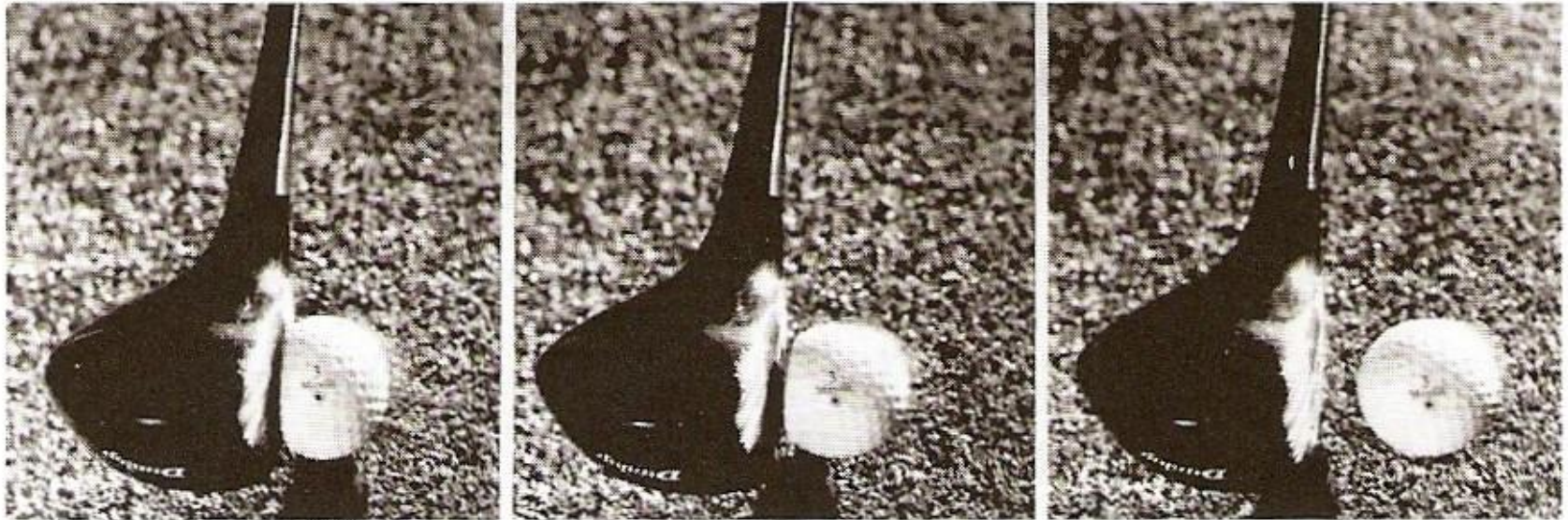
Impulse, Δp

Impulse is equal to the change of momentum produced by a force over a period of time.

$$\begin{aligned} \text{Impulse, } \Delta p &= F\Delta t \\ &= \Delta(mv) \end{aligned}$$

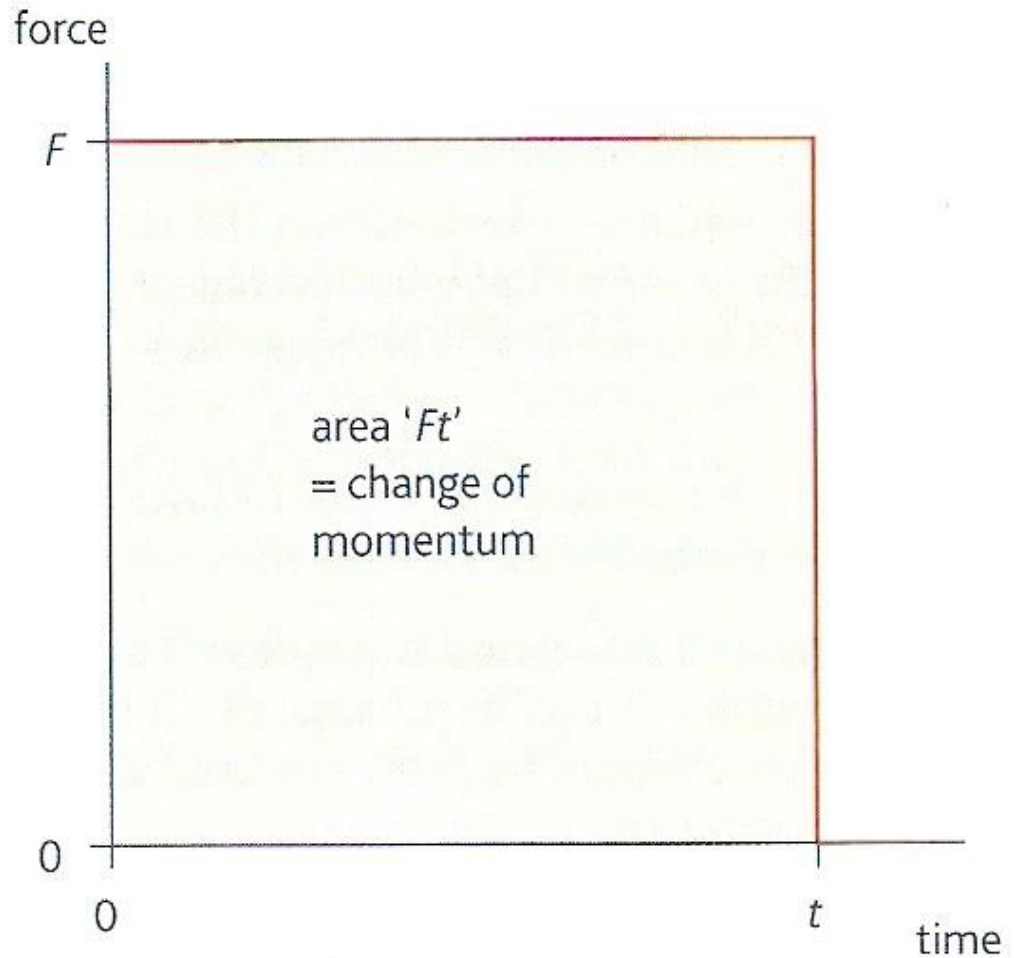
Δp is measured in newton seconds (Ns)

Impulse caused by a golf club

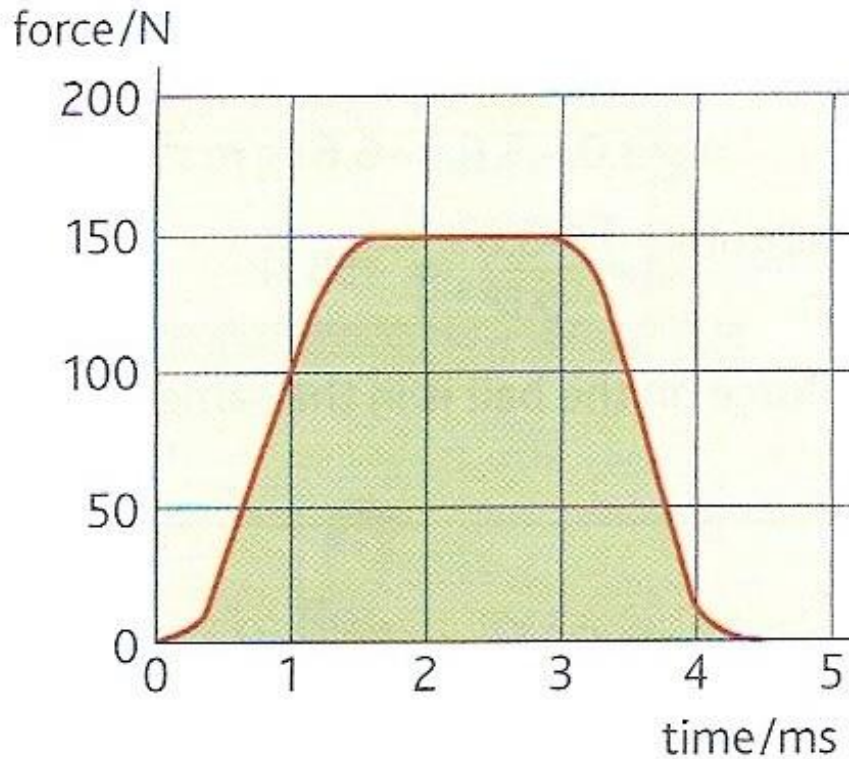


Force – time graphs

Impulse is equal to the area under a force-time graph.



Calculation Example



area under curve = 9 blocks

Ft for 1 block = $50 \text{ N} \times 1 \text{ ms}$

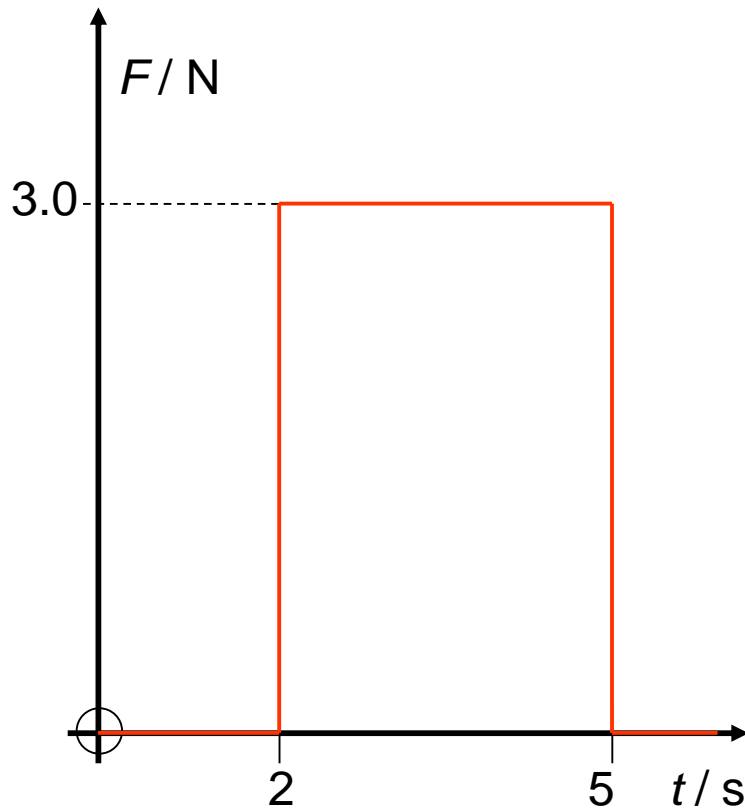
$$= 5.0 \times 10^{-2} \text{ N s}$$

change of momentum

$$= 9 \times 5.0 \times 10^{-2}$$

$$= 0.45 \text{ N s}$$

Graph Question



Calculate the impulse and change in velocity caused to mass of 6kg from the graph opposite.

Area = impulse

$$= 3\text{N} \times (5 - 2)\text{s}$$

impulse = 9 Ns

$$= \Delta(mv) = 6\text{kg} \times \Delta(v)$$

$$\text{therefore, } \Delta(v) = 9 / 6$$

velocity change = 1.5 ms⁻¹

Momentum conservation

Momentum is conserved in any collision or explosion provided no external forces act on the colliding or exploding bodies.



The initial momentum of the yellow car has been conserved and transferred to the red car

Conservation of Linear Momentum

The total linear momentum of an isolated system of bodies remains constant

An isolated system is one where no external forces (e.g. friction or air resistance) acts on the interacting bodies.

Question 1

A trolley of mass 4kg moving at 5ms^{-1} collides with another initially stationary trolley of mass 3kg. If after the collision the trolleys move off attached together calculate their common final velocity.

Initial total linear momentum of the system:

= momentum of 4kg trolley + momentum of 3kg trolley

= $(4\text{kg} \times 5\text{ms}^{-1}) + (3\text{kg} \times 0\text{ms}^{-1})$

= 20 kgms^{-1}

Conservation of linear momentum:

Final total linear momentum of the system

must also = 20 kgms^{-1}

(total mass x final common velocity) = 20 kgms^{-1}

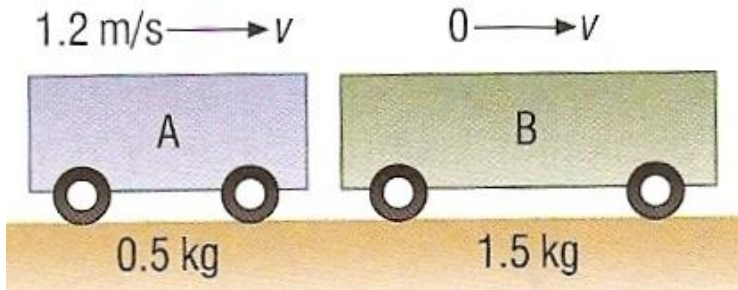
$(4\text{kg} + 3\text{kg}) \times v = 20 \text{ kgms}^{-1}$

$7v = 20$

$v = 20 / 7$

Final common velocity = 2.86 ms^{-1}

Question 2



A truck of mass 0.5kg moving at 1.2m/s collides and remains attached to another, initially stationary truck of mass 1.5kg. Calculate the common velocity of the trucks after the collision.

total momentum before collision

$$p = m \times v$$

$$0.5 \text{ kg truck:} = 0.5 \text{ kg} \times 1.2 \text{ m/s} = 0.6 \text{ kg m/s}$$

$$1.5 \text{ kg truck:} = 1.5 \text{ kg} \times 0 \text{ m/s} = 0 \text{ kg m/s}$$

$$\text{total initial momentum} = 0.6 \text{ kg m/s}$$

Momentum is conserved in the collision

$$\text{so total momentum after collision} = 0.6 \text{ kg m/s}$$

$$\text{total momentum} = \text{total mass} \times \text{velocity}$$

$$0.6 \text{ kg m/s} = 2.0 \text{ kg} \times v$$

$$0.6 \div 2.0 = v$$

$$\text{common velocity} = 0.3 \text{ m/s}$$

Question 3

A train wagon of mass 800 kg moving at 4 m/s collides and remains attached to another wagon of mass 1200 kg that is moving in the same direction at 2 m/s. Calculate the common velocity of the wagons after the collision.

total momentum before collision

$$p = m \times v$$

$$800 \text{ kg wagon:} = 800 \text{ kg} \times 4 \text{ m/s} = 3200 \text{ kg m/s}$$

$$1200 \text{ kg truck:} = 1200 \text{ kg} \times 2 \text{ m/s} = 2400 \text{ kg m/s}$$

$$\text{total initial momentum} = 5600 \text{ kg m/s}$$

Momentum is conserved in the collision

$$\text{so total momentum after collision} = 5600 \text{ kg m/s}$$

$$\text{total momentum} = \text{total mass} \times \text{velocity}$$

$$5600 \text{ kg m/s} = 2000 \text{ kg} \times v$$

$$5600 \div 2000 = v$$

$$\text{common velocity} = 2.8 \text{ m/s}$$

Choose appropriate words to fill in the gaps below:

The momentum of an object is equal to its mass multiplied by its velocity. Momentum has direction, the same as the velocity, and is measured in kilogram metres per second.

In any interaction of bodies, where no external forces act on the bodies, momentum is conserved.

In snooker, a head-on collision of a white ball with a red ball can result in the red ball moving off with the same initial velocity of the white ball. This is an example of momentum conservation.

WORD SELECTION:

direction forces same conservation

metres momentum mass

Elastic and inelastic collisions

ELASTIC – **KINETIC** energy is conserved

INELASTIC – Some (or all) **KINETIC** energy is transformed into thermal or other forms of energy.

In both types of collision both the total energy and momentum are conserved.

Collision question continued

Was the collision in the previous example elastic or inelastic?

Kinetic energy = $\frac{1}{2} \times \text{mass} \times (\text{speed})^2$

Total initial KE = KE of 4kg trolley

$$= \frac{1}{2} \times 4\text{kg} \times (5 \text{ ms}^{-1})^2 = 2 \times 25 = 50 \text{ J}$$

Total final KE = KE of combined 7kg trolley

$$= \frac{1}{2} \times 7\text{kg} \times (2.86 \text{ ms}^{-1})^2 = 3.5 \times 8.18 = 28.6 \text{ J}$$

Kinetic energy reduced – Collision INELASTIC

Explosions



Before an explosion the total momentum is zero.

As momentum is conserved, the total momentum afterwards must also be zero.

This means that the different parts of the exploding body must move off in different directions.

Explosions



KINETIC
energy is
increased

**Both the total
energy and
momentum are
conserved**

Question 1

An artillery gun of mass 1500kg fires a shell of mass 20kg at a velocity of 150m/s. Calculate the recoil velocity of the gun.



NEGATIVE ← **DIRECTION OF MOTION** → **POSITIVE**

The total momentum before and after the explosion is ZERO

$$p = m \times v$$

$$\text{shell:} = 20 \text{ kg} \times +150 \text{ m/s} = +3000 \text{ kg m/s}$$

This must cancel the momentum of the gun.

Therefore the gun's momentum must be -3000 kg m/s

$$\text{gun:} = 1500 \text{ kg} \times \text{recoil velocity} = -3000 \text{ kg m/s}$$

$$\text{recoil velocity} = -3000 \div 1500$$

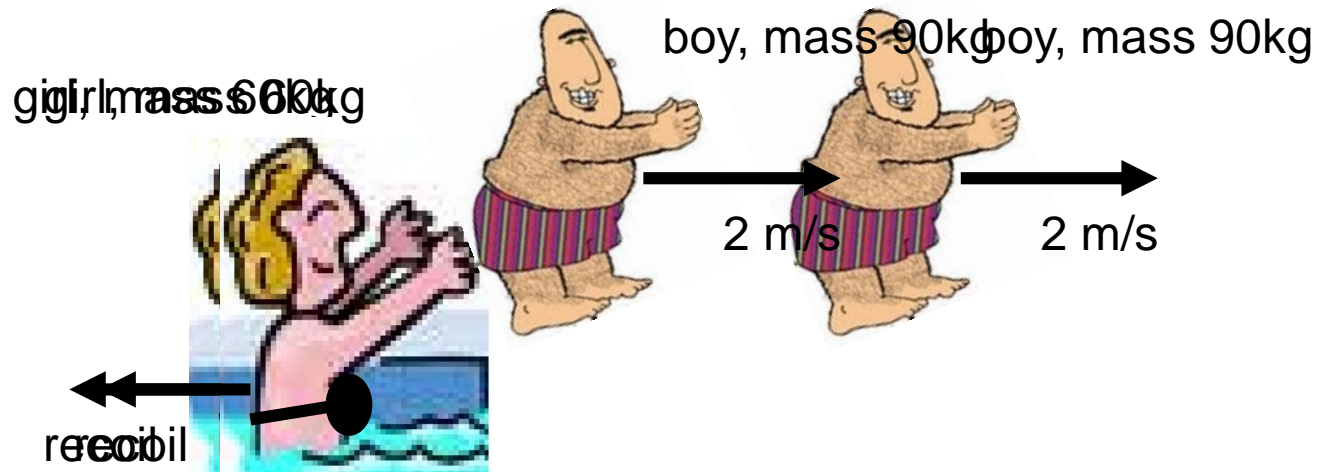
$$= -2 \text{ m/s}$$

The gun will recoil (move to the left)

with a velocity of 2 m/s.

Question 2

A girl of mass 60kg throws a boy, mass 90kg out off a swimming pool at a velocity of 2m/s. What is the girl's recoil velocity?



The total momentum before and after throwing the boy is ZERO

$$p = m \times v$$

$$\text{boy:} = 90 \text{ kg} \times +2 \text{ m/s} = +180 \text{ kg m/s}$$

This must cancel the momentum of the girl.

Therefore the girl's momentum must be -180 kg m/s

$$\text{gun:} = 60 \text{ kg} \times \text{recoil velocity} = -180 \text{ kg m/s}$$

$$\text{recoil velocity} = -180 \div 60$$

$$= -3 \text{ m/s}$$

**The girl will recoil (move to the left)
with a velocity of 3 m/s.**

Question 3

A gun of mass 3kg fires a bullet of mass 15g. If the bullet moves off at a speed of 250ms^{-1} calculate the recoil speed of the gun.

Initial total linear momentum of the system:

= momentum of the gun + momentum of the bullet

= $(3\text{kg} \times 0\text{ms}^{-1}) + (15\text{g} \times 0\text{ms}^{-1})$

= 0 kgms^{-1}

Conservation of linear momentum: Final total linear momentum of the system must also = 0 kgms^{-1}

Therefore:

$$(\text{bullet mass} \times \text{velocity}) + (\text{gun mass} \times \text{velocity}) = 0$$

$$(0.015\text{kg} \times 250\text{ms}^{-1}) + (3\text{kg} \times \text{gun velocity}) = 0$$

$$(3.75) + (3 \times \text{gun velocity}) = 0$$

$$3 \times \text{gun velocity} = - 3.75$$

$$\text{gun velocity} = - 3.75 / 3 = - 1.25 \text{ ms}^{-1}$$

The MINUS sign indicates that the gun's velocity is in the opposite direction to that of the bullet

Gun recoil speed = 1.25 ms⁻¹

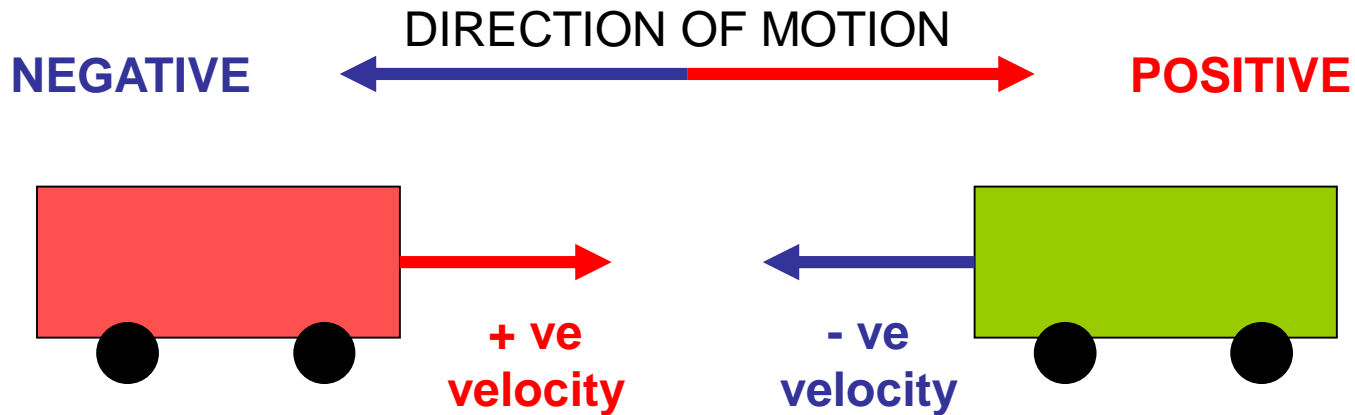
Head-on collisions

In this case bodies are moving in opposite directions.

Momentum has direction.

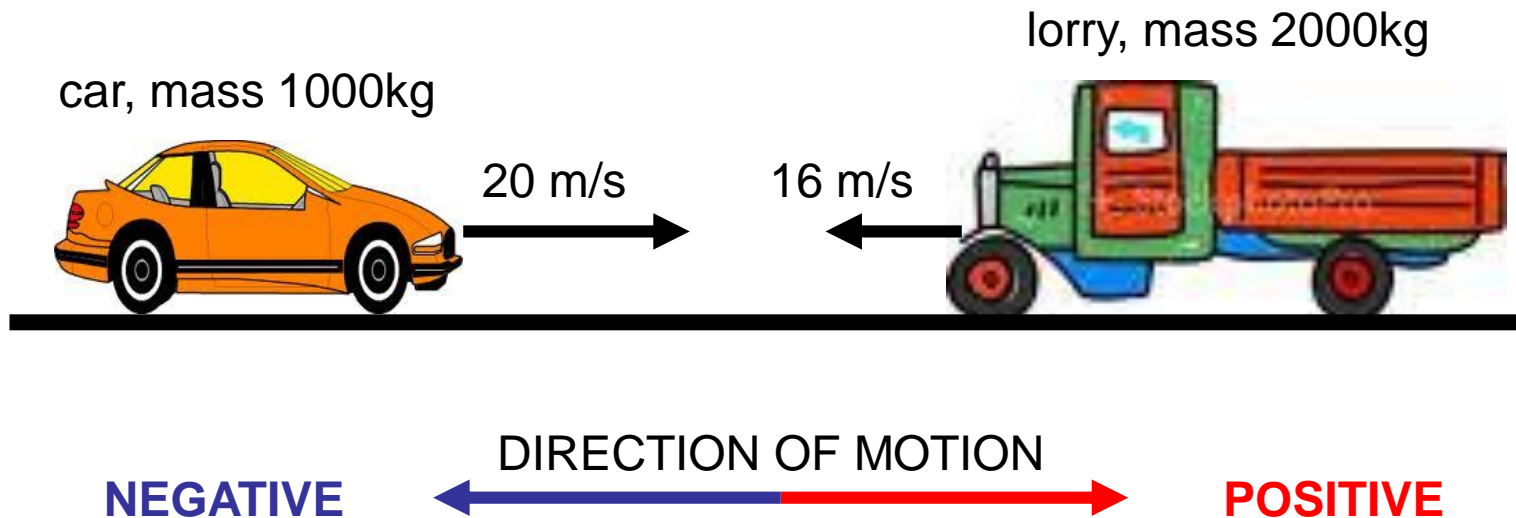
One direction is treated as positive, the other as negative.

In calculations the velocity of one of the colliding bodies must be entered as a **NEGATIVE** number.



Question 1

A car of mass 1000 kg moving at 20 m/s makes a head-on collision with a lorry of mass 2000 kg moving at 16 m/s. Calculate their common velocity after the collision if they remain attached to each other.



total momentum before collision

$$p = m \times v$$

$$\text{car:} = 1000 \text{ kg} \times +20 \text{ m/s} = +20000 \text{ kg m/s}$$

$$\text{lorry:} = 2000 \text{ kg} \times -16 \text{ m/s} = -32000 \text{ kg m/s}$$

$$\text{total initial momentum} = -12000 \text{ kg m/s}$$

Momentum is conserved in the collision

$$\text{so total momentum after collision} = -12000 \text{ kg m/s}$$

$$\text{total momentum} = \text{total mass} \times \text{velocity}$$

$$-12000 \text{ kg m/s} = 3000 \text{ kg} \times v$$

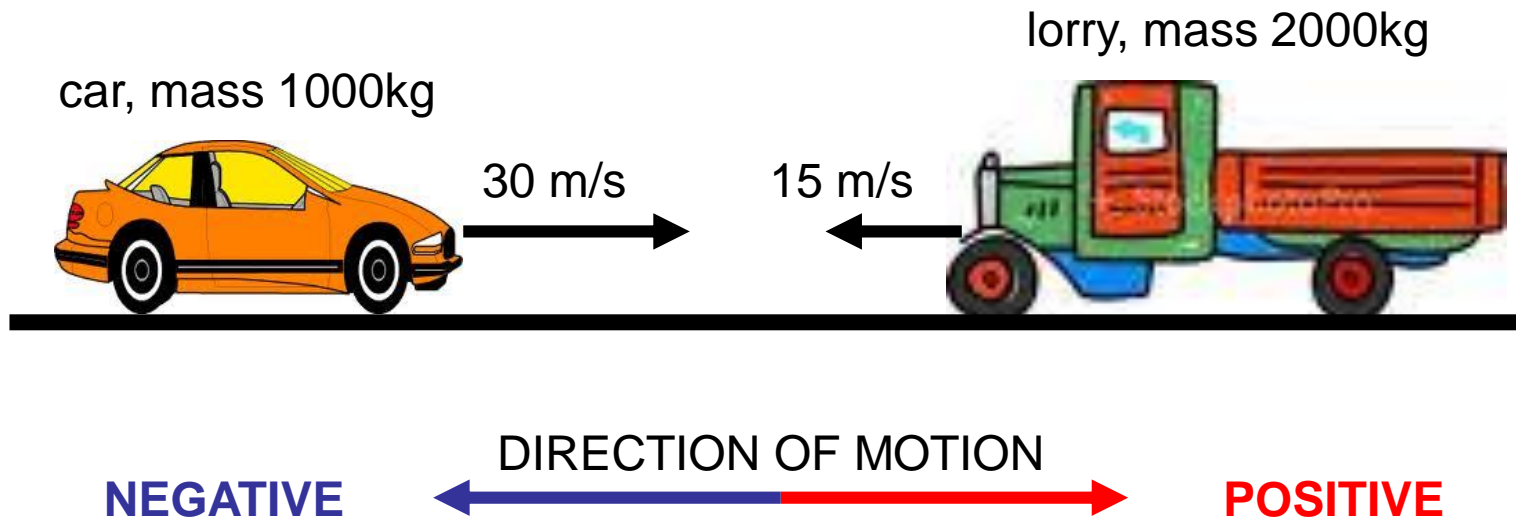
$$-12000 \div 3000 = v$$

$$\text{common velocity} = -4 \text{ m/s}$$

The lorry/car combination will move in the original direction of the lorry.

Question 2

A car of mass 1000 kg moving at 30 m/s makes a head-on collision with a lorry of mass 2000 kg moving at 15 m/s. Calculate their common velocity after the collision if they remain attached to each other.



total momentum before collision

$$p = m \times v$$

$$\text{car:} = 1000 \text{ kg} \times +30 \text{ m/s} = +30000 \text{ kg m/s}$$

$$\text{lorry:} = 2000 \text{ kg} \times -15 \text{ m/s} = -30000 \text{ kg m/s}$$

$$\text{total initial momentum} = 0 \text{ kg m/s}$$

Momentum is conserved in the collision

$$\text{so total momentum after collision} = 0 \text{ kg m/s}$$

The lorry/car combination will not move after the collision.