

Work, Energy and Power

"When a force acts upon an object." Energy can't be created or destroyed, it can only be changed from one type into another type. We call this rule **conservation of energy**.

Work

Work and energy are the same thing. When a force moves something along any distance we say that work has been done and energy has been transformed (energy has been changed from one type to another type). The different kinds of energy that you will have met before are

- **Kinetic**
- **Potential**
- **Light**
- **Sound**
- **Nuclear**
- **Heat**
- **Electrical**
- **Chemical**

For example, if a box is pushed across the floor, work has to be done to overcome the force of friction between the floor and the bottom of the box that is opposing the movement.

How do we calculate the work done?

Energy transformed = work done = applied force x distance

We write this equation as

$$\mathbf{WD = F \times d}$$

Example:

A crate of mass 50kg is pushed along a floor with a force of 20N for a distance of 5m. Calculate the work done.

Solution:

Use

$$WD = F \times d$$

$$= 20\text{N} \times 5\text{m}$$

$$\underline{WD = 100\text{Nm}}$$

But energy transformed = work done

Energy is measured in Joules (J) so Nm must be the same unit as J.

This means we can say that

$$\underline{\underline{WD = 100\text{J}}}$$

Note that the mass of the crate is not required to answer the question. The mass would be required if we needed to calculate the work done by *lifting* the crate but not by pushing it.

Example:

How far must a 5N force pull a 50g toy car if 30J of energy are transferred?

Solution:

Use

$$WD = F \times d$$

Rearrange to get

$$d = \frac{WD}{F} = \frac{30}{5}$$

so $\underline{d = 6\text{m}}$

Example:

A man exerts a force of 2kN on a boulder but fails to move it. Calculate the work done.

Solution:

Use

$$\begin{aligned} \text{WD} &= F \times d && d = 0 \text{ because the boulder does} \\ &= 2000 \times 0 && \text{not move} \end{aligned}$$

so

$$\underline{\text{WD} = 0}$$

If an object does not move when the force is applied then no work is done. Work is only done if the object moves.

Gravitational Potential Energy

Whenever we lift up an object we are providing a force to act against gravity. By lifting the object we are storing energy in it. We can calculate the energy stored in an object when it is raised up by looking at the work done in lifting it.

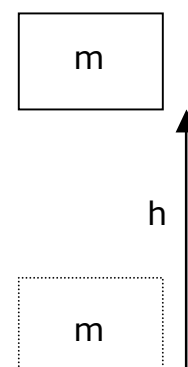
Imagine that a box of mass m is lifted up to a height h above the ground.

The force that must be overcome to move the box is the weight due to gravity. The weight (W) is given by

$$W = \text{mass} \times \text{gravitational field strength}$$

So

$$W = mg \text{ (= Force to lift the box)}$$



Now we can use $WD = F \times d$ to calculate the stored energy.

$$\begin{aligned}WD &= F \times d \\ &= (mg) \times d \\ &= (mg) \times h\end{aligned}$$

so

$$\mathbf{WD = mgh}$$

This tells us that the energy needed to lift something (E) is given by

$$E = \text{mass} \times \text{gravitational field strength} \times \text{height}$$

In other words, the potential energy **stored** in the object by raising it up is given by

$$\mathbf{E_p = mgh}$$

Example:

A football of mass 2.5kg is lifted up to the top of a cliff that is 180m high. How much potential energy does the football gain?

Solution:

Use

$$\begin{aligned}E_p &= mgh \\ &= 2.5 \times 10 \times 180\end{aligned}$$

so

$$\underline{E_p = 4500J}$$

Power

Power is the way we measure how quickly energy is being changed. When we look at the power of a moving object, we are really looking at how fast work is happening.

We define power as

$$\text{Power} = \frac{\text{work done}}{\text{time taken}}$$

and using symbols, we write this as

$$P = \frac{(F \times d)}{t}$$

Power is measured in **Watts**.

Example:

A person of mass 70kg runs up a flight of stairs with a vertical height of 5m. If the trip takes 7s to complete, calculate the person's power.

Solution:

$$\begin{aligned} \text{WD} &= E_p = mgh \\ &= 70 \times 10 \times 5 \\ &= 3500\text{J} \end{aligned}$$

$$\text{Power} = \frac{\text{work done}}{\text{time}} = \frac{3500}{7}$$

so

$$\underline{\text{Power} = 500\text{W}}$$

Example:

A lift motor has to move a fully laden lift 4m between floors in 1.5s. The lift has a mass of 1850kg (ignore friction).

a) Calculate the weight of the fully laden lift.

$$W = mg = 1850 \times 10 = \underline{18500N}$$

b) What is the upward force in the cable when the lift is moving at a constant speed?

At constant speed, forces must be balanced.

Upward force = downward force (weight)

$$\underline{\text{Upward force} = 18500N}$$

c) What is the work done by the motor?

$$WD = F \times d$$

$$= 18500 \times 4$$

so

$$\underline{WD = 74000J}$$

d) What is the minimum power of the motor to raise the lift at a steady speed?

Steady speed means forces are balanced.

$$\text{Power} = \frac{\text{work done}}{\text{time}} = \frac{74000}{1.5}$$

so

$$\underline{\text{Power} = 49333W}$$

Kinetic Energy

Any moving object has Kinetic Energy (E_k).

- The kinetic energy of an object will increase if mass increases.
- The kinetic energy will increase if speed increases.

We can calculate kinetic energy using the equation

$$E_k = \frac{1}{2}mv^2$$

where m is the mass of the object and v is the speed.

Notice that the speed is squared in the equation. This means that

- if speed is doubled, energy $\times 4$
- if speed is trebled, energy $\times 9$

Example:

How much kinetic energy has a 160g cricket ball when it is thrown at a speed of 22m/s?

Solution:

use

$$E_k = \frac{1}{2}mv^2$$

$$= 0.5 \times 0.16 \times (22)^2$$

$$= 0.5 \times 0.16 \times 484$$

so

$$\underline{E_k = 38.72J}$$

Example:

How fast is a trolley moving if it has 180.5J of kinetic energy?

Solution:

$$E_k = \frac{1}{2}mv^2$$

rearrange to get

$$2E_k = mv^2$$

so

$$v^2 = \frac{2E_k}{m}$$

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 180.5}{4}}$$

$$= \sqrt{\frac{391}{4}}$$

$$= \sqrt{90.25}$$

so $v = 9.5\text{m/s}$

Potential and Kinetic Energy often appear in the same problems. Imagine a brick falling from a wall. The brick originally has only potential energy. As the brick falls, it loses potential energy but gains kinetic energy as it accelerates towards the ground.

The total energy of the brick at any point in time is given by the sum of the potential and kinetic energies and is always equal to the initial potential energy. This is due to the conservation of energy.

Example:

A ball rolls off a table and hits the floor at 5m/s. What is the height of the table.

Initial energy = E_p

Final energy = E_k

But conservation of energy tells us $E_p = E_k$.

$$\text{So } mgh = \frac{1}{2}mv^2$$

$$2gh = v^2$$

so

$$h = \frac{v^2}{2g} = \frac{25}{20}$$

so

$$\underline{h = 1.25\text{m}}$$