VECTORS

Distance and Displacement.

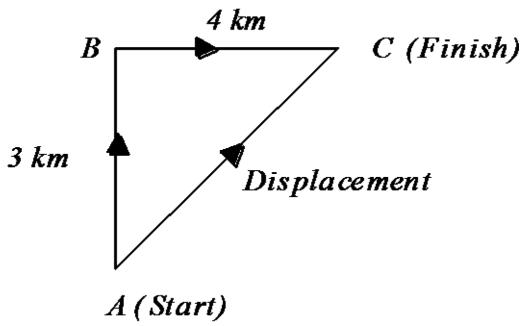
Distance is the total path length. It is fully described by magnitude (size) alone. Displacement is the direct length from a starting point to a finishing point. To describe displacement **both** magnitude **and** direction must be given.

Example

A woman walks 3 km due North (000) and then 4 km due East (090).

Find her a) distance travelled

b) displacement i.e. how far she is from where she started ?



Using a scale of 1cm: 1km draw an accurate scale diagram as shown above.

a) Distance travelled = AB + BC = 7 km

b) Measuring AC = 5 cm.

Convert using the scale gives the magnitude of the displacement = 5 km

Use a protractor to check angle BAC = 53^o that is 53^o east of north.

Displacement = 5 km (053)

Speed and Velocity

These two quantities are fundamentally different.

$$\text{Average speed} = \frac{\text{total distance}}{\text{time}} \qquad \text{Average velocity} = \frac{\text{total displacement}}{\text{time}}$$

Velocity has an associated direction, being the same as that of the displacement.
The unit for both these quantities is metres per second, m s^{-1}.

Vectors and Scalars

A **scalar** quantity is completely defined by stating its **magnitude**.
A **vector** quantity is completely defined by stating its **magnitude** and **direction**.
Examples are given below.

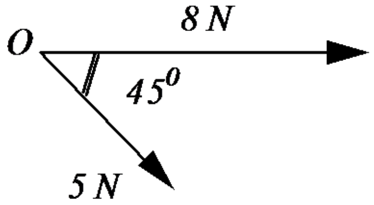
Vectors	Scalars
Displacement	Distance
Velocity	Speed
Acceleration	Time
Force	Mass
Impulse	Energy

Addition of Vectors

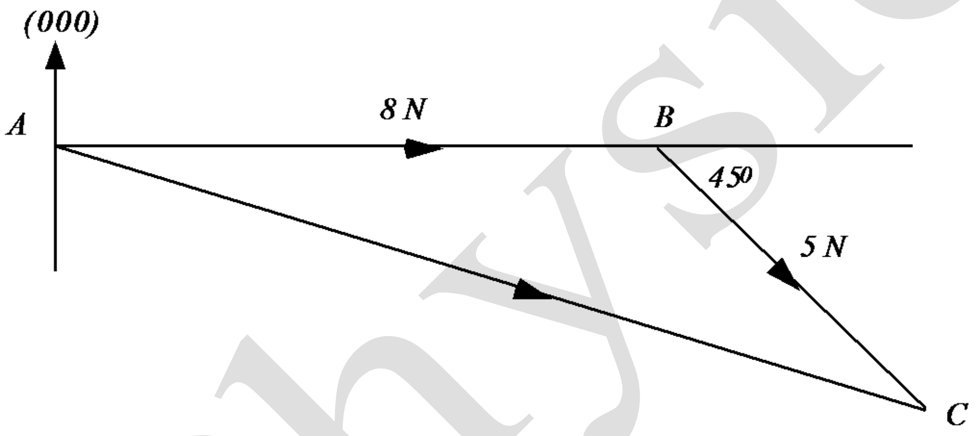
When vectors are being added, their magnitude and direction must be taken into account. This can be done using a scale diagram and adding the vectors 'tip to tail', then joining the starting and finishing points. The final sum is known as the resultant, the single vector that has the same effect as the sum of the individuals.
Another important example of a vector is a force.

Example

Find the resultant force acting at point O.



Step 1: Choose a suitable scale, e.g. 1 cm to 1 N.
Step 2: Arrange arrows "tip to tail". Choose either one, slide other into position, tip to tail.
Step 3: Draw in resultant vector, measuring its length and direction.



1 cm to 1 N.	AC	=	12.2 cm
	Force	=	12.2 N
			Using a protractor, angle BAC measures 12^0
	Bearing	=	$90^0 + 12^0 = 102^0$
	Resultant Force	=	12.2 N at (102)

Vectors at right angles

If the vectors are at right angles then it may be easier to use Pythagoras to find the resultant and trigonometry to find an angle. If not, sine or cosine rule can be used.

Addition of more than two vectors

Use a scale diagram and ensure that each vector is placed "tip to tail" to the previous vector. The resultant vector is the vector from the starting point to the finishing point in magnitude and direction.

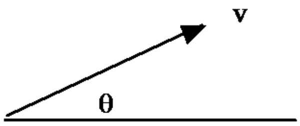
Resultant of a number of forces – FORCE IS A VECTOR

The resultant of a number of forces is that single force which has the same effect, in both magnitude and direction, as the sum of the individual forces.

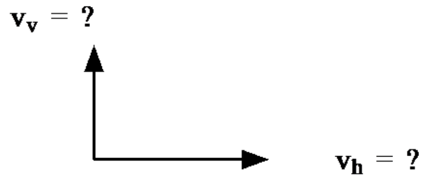
Rectangular components of a vector

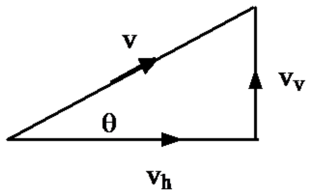
Resolution of a vector into horizontal and vertical components.

Any vector v can be split up into a horizontal component v_h and vertical component v_v.



is equivalent to





$$\sin\theta = \frac{V_v}{v}$$

$$\mathbf{v_v = v \sin\theta}$$

$$\cos\theta = \frac{V_h}{v}$$

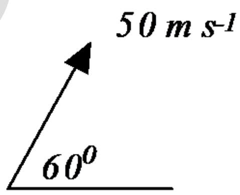
$$\mathbf{v_h = v \cos\theta}$$

Example

A shell is fired from a cannon as shown.

Calculate its a) horizontal component of velocity

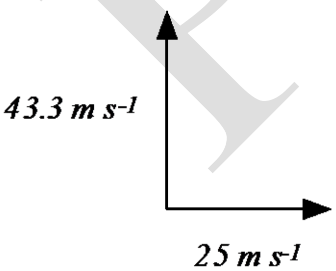
b) vertical component of velocity.



a) $v_h = v \cos u = 50 \cos 60° = 25\ m\ s^{-1}$

b) $v_v = v \sin u = 50 \sin 60° = 43\ m\ s^{-1}$

So



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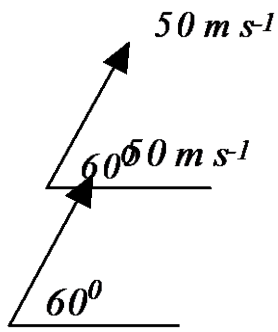


EQUATIONS OF MOTION

Acceleration

Acceleration is defined as the change in velocity per unit time. It is a VECTOR
The unit is metre per second squared, m s^{-2}.

$$a = \frac{v - u}{t}$$

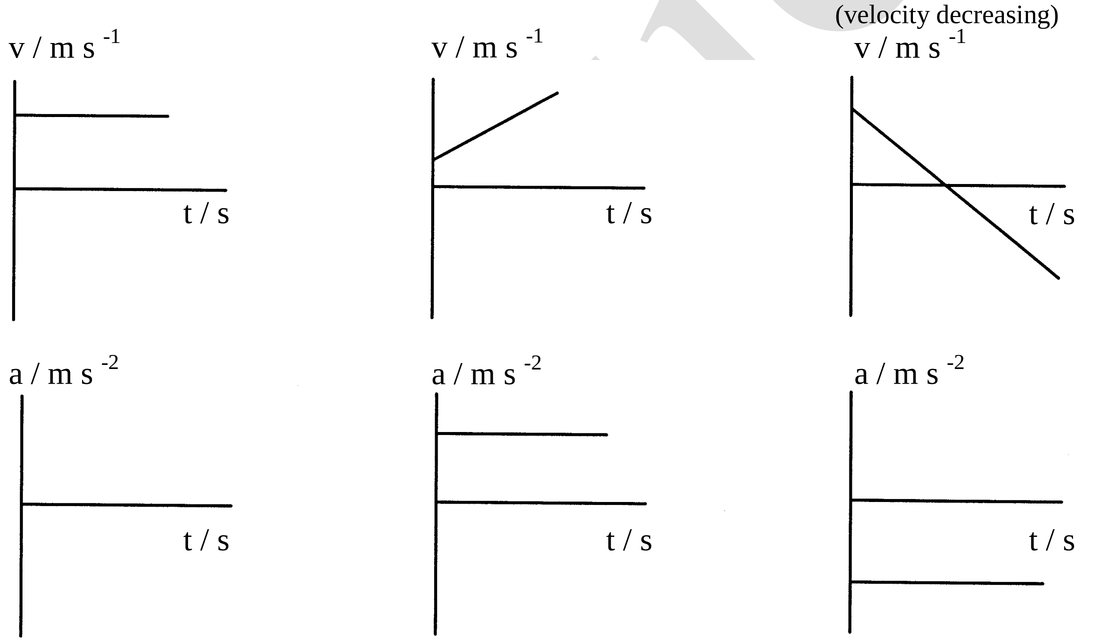
where v = final velocity
u = initial velocity
t = time taken

Measuring acceleration

Acceleration is measured by determining the initial velocity, final velocity and time taken. A double mask which interrupts a light gate can provide the data to a microcomputer and give a direct reading of acceleration.

Acceleration-time and velocity-time graphs

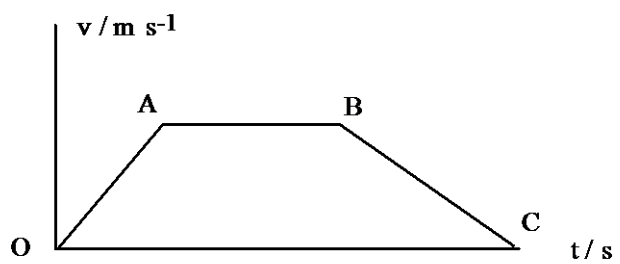
Constant velocity | Constant positive acceleration (velocity increasing) | Constant deceleration Constant negative acceleration (velocity decreasing)



a = 0 | acceleration is positive | acceleration is negative

Constant velocity and constant acceleration

The velocity time graph below illustrates these terms.



OA is constant acceleration,
the acceleration is positive.
AB is constant velocity,
the acceleration is zero.
BC is constant deceleration,
the acceleration is negative.

Equations of motion

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

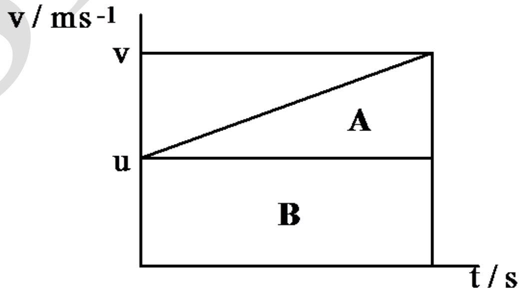
where:
- u - initial velocity of object at time t = 0
- v - final velocity of object at time t
- a - acceleration of object
- t - time to accelerate from u to v
- s - displacement in time t.

These equations of motion apply providing:
- the motion is in a straight line
- the acceleration is uniform.

When using the equations of motion, **note**:
- the quantities u, v, s and a are all vector quantities
- a positive direction must be chosen and quantities in the reverse direction must be given a negative sign e.g. if up is +ve, down is –ve.
- a deceleration will be negative, for movement in the positive direction.

Derivation of equations of motion

The velocity - time graph for an object accelerating uniformly from u to v in time t is shown below.



$$a = \frac{v - u}{t}$$

Changing the subject of the formula gives:

$$v = u + at \qquad \text{---------} \quad [1]$$

The displacement s in time t is equal to the area under the velocity time graph.

Area = Area of triangle A + area of rectangle B

$$s = \frac{1}{2}(v-u)t + ut$$

but from equation 1, $v-u = At$

$$s = \frac{1}{2}(at)t + ut$$

$$s = ut + \frac{1}{2}at^2 \qquad \text{---------} \quad [2]$$

Using v = u + at,

$$v^2 = (u + at)^2$$

$$v^2 = u^2 + 2uat + a^2t^2$$

$$v^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$$

Since $s = ut + \frac{1}{2}at^2$

$$v^2 = u^2 + 2as \qquad \text{---------} \quad [3]$$

Projectile motion

A projectile has a combination of vertical and horizontal motions. Various experiments show that these horizontal and vertical motions are totally independent of each other.
Closer study gives the following information about each component.

Horizontal: **constant speed**

Vertical: **constant acceleration downward (due to gravity).**

Example

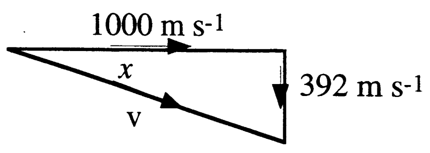
An object is released from an aircraft travelling horizontally at 1000 m s^{-1}. The object takes 40 s to reach the ground.

a) What is the horizontal distance travelled by the object?
b) What was the height of the aircraft when the object was released?
c) Calculate the vertical velocity of the object just before impact.
d) Find the resultant velocity of the object just before hitting the ground.

Before attempting the solution, you should divide your page into horizontal and vertical and enter appropriate information given or known.

Horizontal	***Vertical***
$v_h = 1000 \text{ m s}^{-1}$ $\quad t = 40 \text{ s}$	$t = 40 \text{ s}$ $\quad u_v = 0$ $\quad a = 9.8 \text{ m s}^{-2}$
a) $s_h = ?$	b) $s_v = ?$ $\quad s_v = ut + \frac{1}{2} at^2$
$s_h = v \times t = 1000 \times 40 = 40000 \text{ m}$	$= 0 + \frac{1}{2} \times 9.8 \times 40^2$
	$= 7840 \text{ m}$
	c) $v_v = ?$
	$v_v = u + at = 0 + 9.8 \times 40$
	$= 392 \text{ m}$
	$v_v = 392 \text{ m s}^{-1}$ (downwards)

d)



$$\tan x = \frac{392}{1000} \qquad x = 21^o$$

$$v^2 = 1000^2 + 392^2$$
$$v = 1074 \text{ m s}^{-1}$$

Resultant velocity $= 1074 \text{ m s}^{-1}$ *at (111)*

NEWTON'S SECOND LAW, ENERGY AND POWER

Dynamics deals with the forces causing motion and the properties of the resulting moving system.

Newton's 1st Law of Motion LEARN!

Newton's 1st law of Motion states that an object will remain at rest or travel with a constant speed in a straight line (constant velocity) unless acted on by an <u>unbalanced</u> force.

Newton's 2nd Law

Newton's 2nd law of motion states that the acceleration of an object:

- varies directly as the <u>unbalanced</u> force applied if the mass is constant
- varies inversely as the mass if the unbalanced force is constant.

These can be combined to give

$$a \propto \frac{F}{m}$$

$$a = \frac{kF}{m} \quad \text{where k is a constant}$$

$$kF = ma$$

LEARN THIS! The unit of force, the **newton** is defined as the resultant force which will cause a mass of 1kg to have an acceleration of 1 m s^{-2}. Substituting in the above equation,

$$k \times 1 = 1 \times 1$$
$$k = 1$$

Provided F is measured in newtons, the equation below applies.

$$\boxed{F = ma}$$

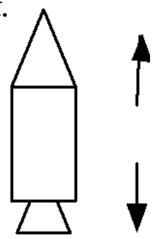
F: N; m: kg; a: m s^{-2}

Free Body Diagrams

Some examples will have more than one force acting on an object. It is advisable to draw a diagram of the situation showing the direction of all forces present acting through one point. These are known as free body diagrams.

Examples

1. *On take off, the thrust on a rocket of mass 8000 kg is 200,000 N. Find the acceleration of the rocket.*



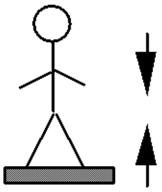
Thrust = 200,000 N

Weight = mg = 8000 × 9.8 = 78,400 N

Resultant force = 200000 - 78,400 = 121,600 N <u>*upwards*</u>

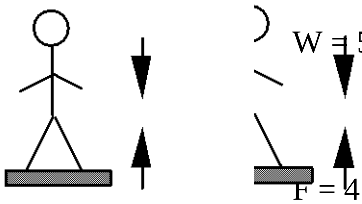
$$a = \frac{F}{m} = \frac{121600}{8000} = 15.2 \text{ m s}^{-2} \text{ \underline{up}}$$

2. *A woman is standing on a set of bathroom scales in a stationary lift (a normal everyday occurrence!). The reading on the scales is 500 N. When she presses the ground floor button, the lift accelerates downwards and the reading on the scales at this moment is 450 N. Find the acceleration of the lift.*



Weight = 500 N

Force upwards = 500 N



W = 500 N

F = 450 N
(reading on scales)
(Scales read reaction upwards)

Lift is stationary, forces balance

W = F

= 500 N

Lift accelerates downwards,
unbalanced force acts.

Resultant Force = Weight - Force from floor

= W - F

= 500 – 450

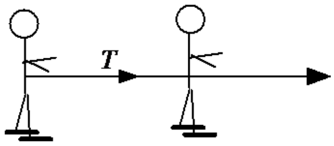
= 50 N

$$a = \frac{\text{Resultant Force}}{m}$$

$$= \frac{50}{50}$$

$= 1 \text{ m s}^{-2}$

3. *Tension*
A ski tow pulls 2 skiers who are connected by a thin nylon rope along a frictionless surface. The tow uses a force of 70 N and the skiers have masses of 60 kg and 80 kg. Find a) the acceleration of the system
b) the tension in the rope.



60 kg 80kg

a) *Total mass, m = 140 kg*

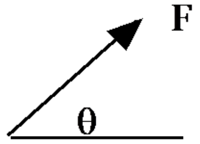
$$a = \frac{F}{m} = \frac{70}{140} = 0.5 \text{ m s}^{-2}$$

b) *Consider the 60 kg skier alone.*

Tension, T = ma = 60 × 0.5 = 30 N

Resolution of a Force

In the previous section, a vector was split into horizontal and vertical components. This can obviously apply to a force.



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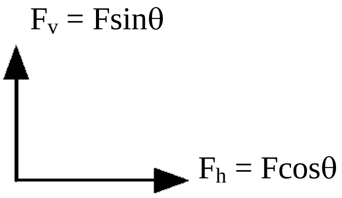
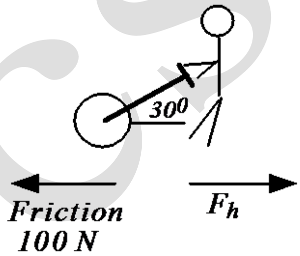


Example

A man pulls a garden roller of mass 100 kg with a force of 200 N acting at 30^0 to the horizontal. If there is a frictional force of 100 N between the roller and the ground, what is the acceleration of the roller along the ground?



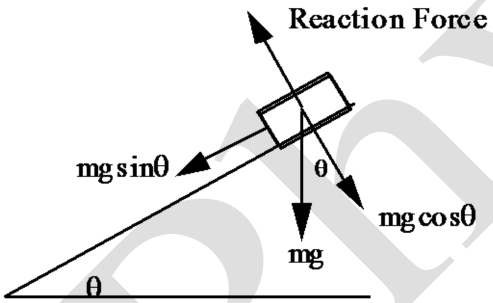
$F_h = F\cos\theta = 200\cos 30^0 = 173.2\ N$

Resultant $F_h = 173.2 - \text{Friction} = 173.2 - 100 = 73.2\ N$

$$a = \frac{F}{m} = \frac{73.2}{100} = 0.732\ m\ s^{-2}$$

Force Acting Down a Plane

If an object is placed on a slope then its weight acts vertically downwards. A certain component of this force will act down the slope. The weight can be split into two components at right angles to each other.

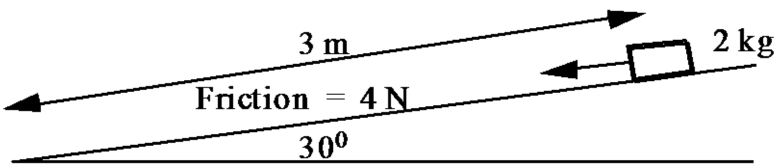


Component of weight down slope = mgsinθ

Component perpendicular to slope = mgcosθ

Example

A wooden block of mass 2 kg is placed on a slope at 30° to the horizontal as shown. A frictional force of 4 N acts up the slope. The block slides down the slope for a distance of 3 m. Determine the speed of the block at the bottom of the slope.



Component of weight acting down slope = mg sin30° = 2 × 9.8 × 0.5 = 9.8 N

Resultant force down slope = 9.8 - friction = 9.8 - 4 = 5.8 N

$a = F/m$
$= 5.8 / 2$
$= 2.9\ m\ s^{-2}$

$v^2 = u^2 + 2as$
$= 0 + 2 \times 2.9 \times 3$
$= 17.4$
$v = 4.2\ m\ s^{-1}$