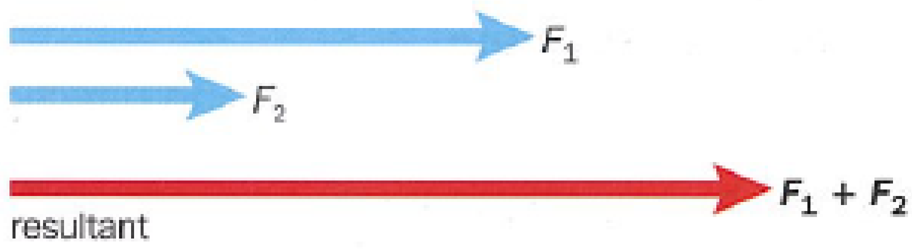
Year 11 Physics

Week 2 Vector Examples

Vector Calculation

Vectors acting along the same straight line

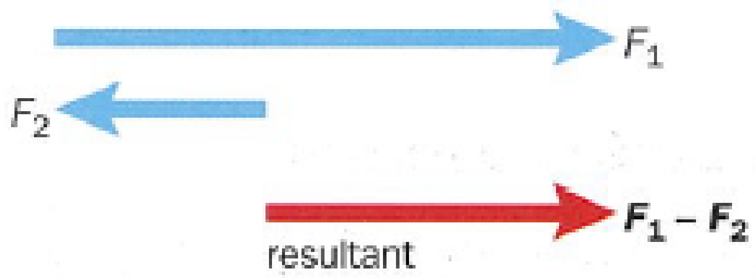
Two vectors acting in the ***same*** direction can simply be added together:



$$\textbf{Resultant} = F_1 + F_2$$

Vector Calculation

If the vectors act in ***opposite*** directions, we need to take one direction as positive, and the other as negative, before adding them:



$$\textbf{Resultant} = F_1 + (-F_2) = F_1 - F_2$$

Vector Example 1

Phiz is standing on a moving walkway in an airport.
The walkway is moving at a steady velocity of $1.50\ \text{m s}^{-1}$.

Phiz starts to walk forwards along the walkway at $2.00\ \text{m s}^{-1}$.
What is his resultant velocity?

Both velocity vectors are acting in the same direction.

Resultant velocity $= 1.50\ \text{m s}^{-1} + 2.00\ \text{m s}^{-1}$

$= \underline{3.50\ \text{m s}^{-1}}$ in the direction of the walkway.

Vector Example 2

Phiz then decides he is going the wrong way. He turns round and starts to run at 3.40 m s^{-1} in the opposite direction to the motion of the walkway. What is his new resultant velocity?

The velocity vectors now act in opposite directions.

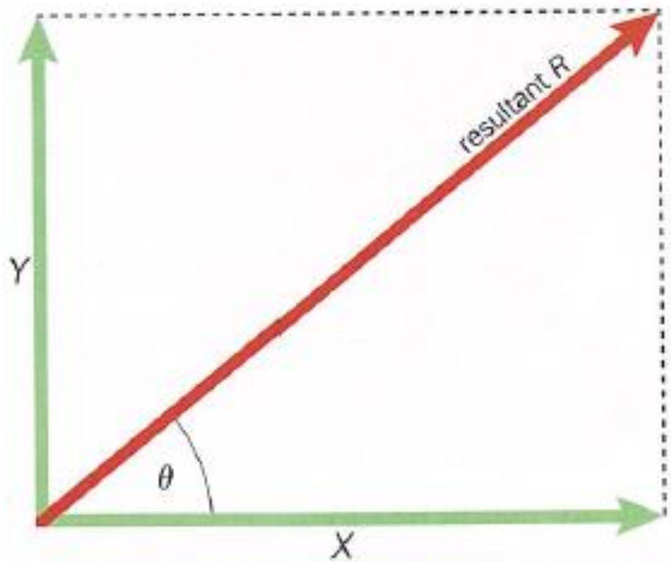
Taking motion in the direction of the walkway to be positive:

$$\text{Resultant velocity} = +1.50 \text{ m s}^{-1} - 3.40 \text{ m s}^{-1}$$
$$= \underline{-1.90 \text{ m s}^{-1}} \quad (3 \text{ s.f.})$$

As this is negative, the resultant velocity acts in the opposite direction to the motion of the walkway. He moves to the left.

Perpendicular vectors

To find the resultant of two vectors (X, Y) acting at 90° to each other, we draw the vectors as adjacent sides of a rectangle:



The resultant is the ***diagonal*** of the rectangle, as shown here:

Vector Calculation: Calculation

The ***magnitude*** (size) of the resultant vector R can be found using Pythagoras' theorem:

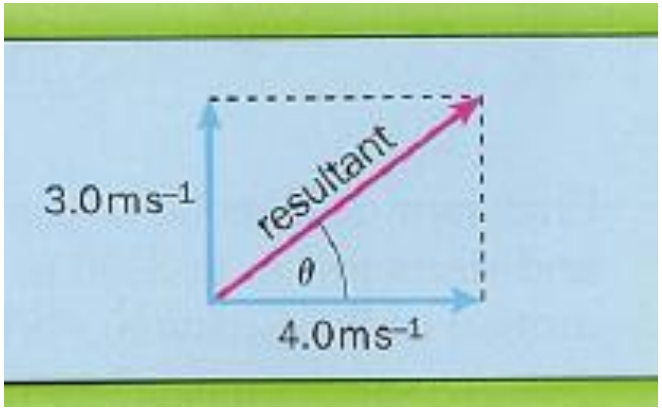
$$\boldsymbol{R}^2 = \mathbf{X}^2 + \mathbf{Y}^2$$

The ***direction*** of the resultant is given by the angle θ:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{Y}{X} \qquad \therefore\ \theta = \tan^{-1}\left(\frac{Y}{X}\right)$$

Worked Example

A man tries to row directly across a river.
He rows at a velocity of 3.0 m s^{-1}.
The river has a current of velocity 4.0 m s^{-1} parallel to the banks.
Calculate the resultant velocity of the boat.



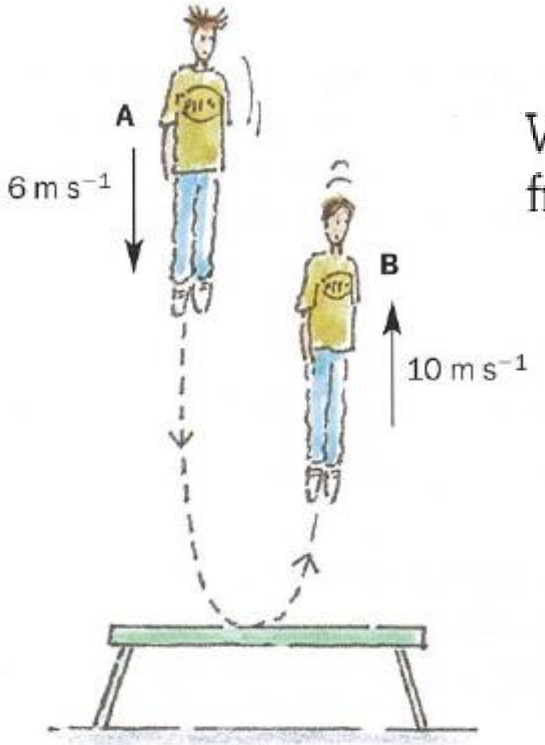
Size of resultant $= \sqrt{(3.0\ \text{m s}^{-1})^2 + (4.0\ \text{m s}^{-1})^2} = \sqrt{25}\ \text{m s}^{-1} = 5.0\ \text{m s}^{-1}$

Direction of resultant: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{3.0}{4.0} \quad \therefore\ \theta = \tan^{-1}\left(\dfrac{3.0}{4.0}\right) = 37°$

So the resultant velocity is 5.0 m s^{-1} at 37° to the bank.

Vector Subtraction

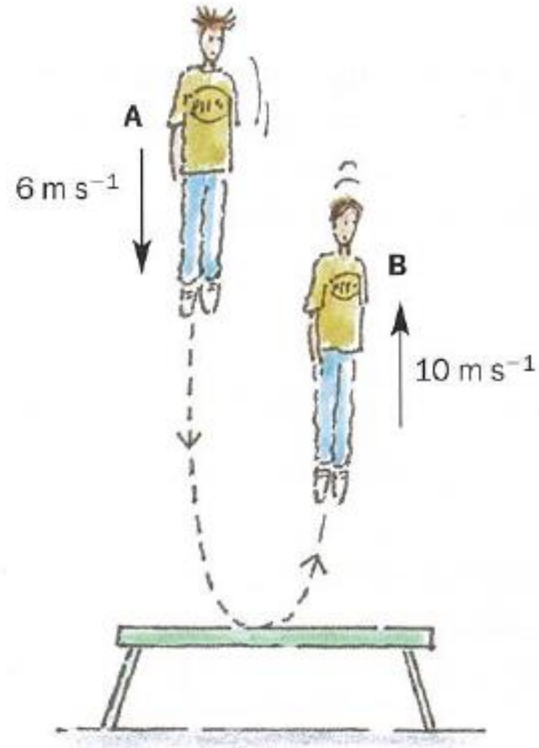
The diagram shows the speed and direction of a trampolinist at two points during a bounce:



What is the trampolinist's change in **speed** from A to B?

Vector Subtraction continued

Change in speed = new speed – old speed
$= 10\ \text{m s}^{-1} - 6\ \text{m s}^{-1}$
$= 4\ \text{m s}^{-1}$



What about his change in **velocity**?

To find the change in a vector quantity we use vector subtraction:

Change in velocity = new velocity – old velocity
$= 10\ \text{m s}^{-1}$ ***up*** $- 6\ \text{m s}^{-1}$ ***down***

Remember, with vectors we must take account of the direction.

Vector Subtraction continued

We can then rewrite our equation as:

$$\text{Change in velocity} = +10\ \text{m s}^{-1} - (-6\ \text{m s}^{-1})$$

$$= +10\ \text{m s}^{-1} + 6\ \text{m s}^{-1}$$

$$= +16\ \text{m s}^{-1}$$

So the change in velocity is 16 m s^{-1} in an ***up***ward direction.

Can you see that subtracting 6 m s^{-1} downwards is the same as adding 6 m s^{-1} acting upwards?

Vector subtraction is the same as the addition of a vector of the same size acting in the opposite direction.

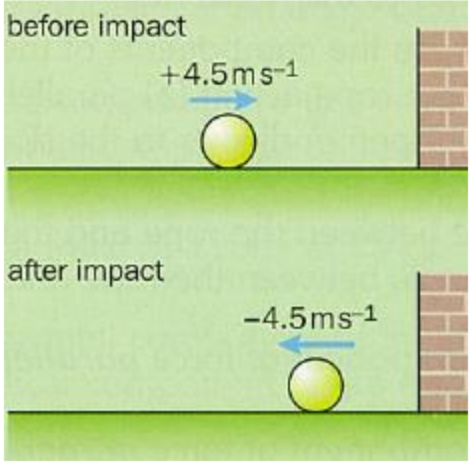
Vector Subtraction Example

A boy kicks a ball against a wall with a horizontal velocity of 4.5 m s^{-1}. The ball rebounds horizontally at the same speed.

What is the ball's change in velocity?

Although the speed is the same, the velocity has changed. Why?

Change in velocity = new velocity – old velocity



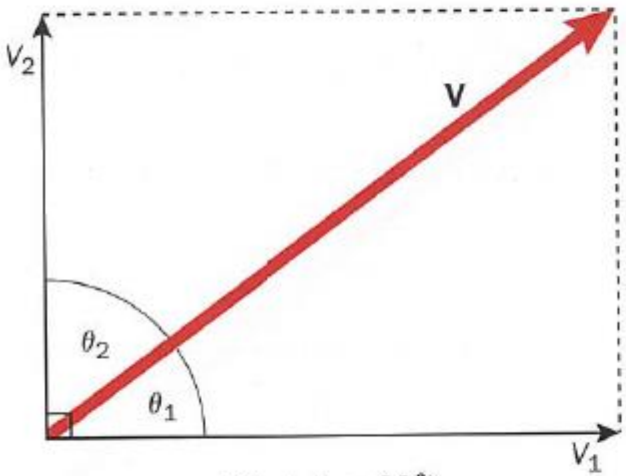
$= (-4.5 \text{ m s}^{-1}) - (+4.5 \text{ m s}^{-1})$

$= -4.5 \text{ m s}^{-1} - 4.5 \text{ m s}^{-1}$

$= -9.0 \text{ m s}^{-1}$ (2 s.f.)

change in velocity is 9.0 m s^{-1} in a direction away from the wall.

Resolving Vectors



$(\theta_1 + \theta_2 = 90°)$

So to find the component of a vector in any direction you need to ***multiply by the cosine of the angle between the vector and the component direction.***

We can resolve this vector into two components, V_1 and V_2, at right angles to each other:

V_1 acts at an angle θ_1 to the original vector.
V_2 acts at an angle θ_2 to the original vector.

To find the size of V_1 and V_2 we need to use trigonometry:

$$\cos\theta_1 = \frac{adjacent}{hypotenuse} = \frac{V_1}{V}$$

Rearranging this gives: $\mathbf{V_1 = V\cos\theta_1}$

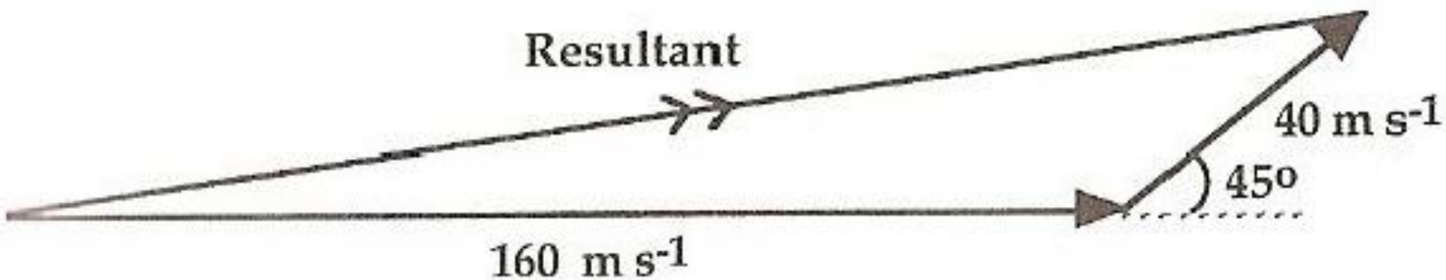
$$\cos\theta_2 = \frac{adjacent}{hypotenuse} = \frac{V_2}{V}$$

Rearranging this gives: $\mathbf{V_2 = V\cos\theta_2}$

Worked Example 3

What is the resultant velocity, relative to the Earth, of a plane which is travelling at 160 m s^{-1} due east, relative to a wind of 40 m s^{-1} from the south-west?

Draw a sketch diagram of the situation.



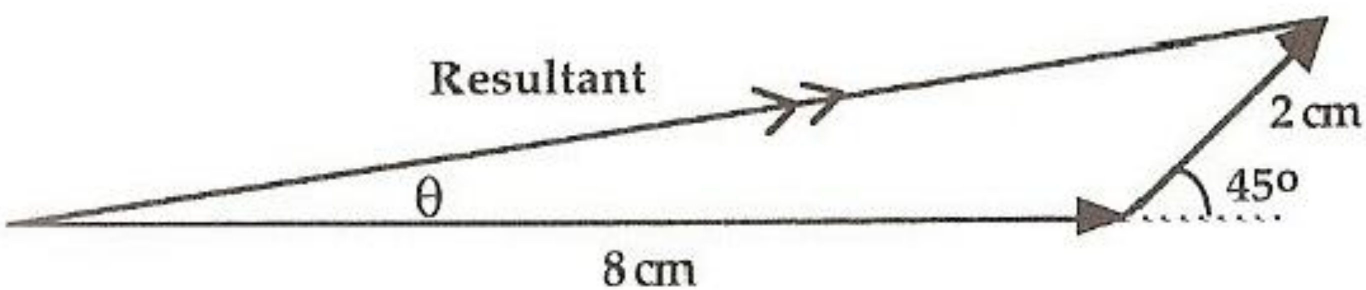
Decide from the sketch whether the problem can best be solved by calculation or if a scale diagram is required. If a scale diagram, then choose a suitable scale.

Worked Example 3 continued

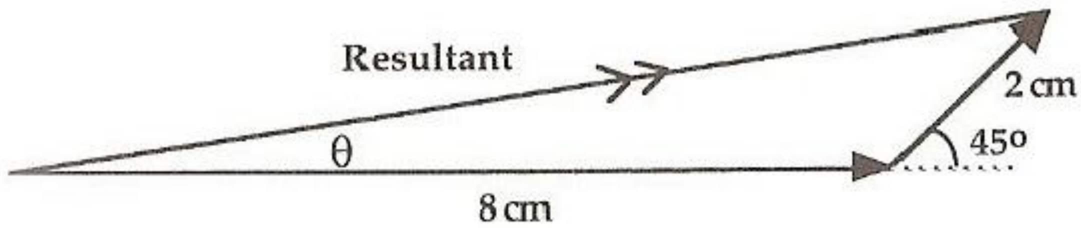
In this case drawing a scale diagram would be easier, since the two vector quantities are not at right angles to each other. A suitable scale might be:

1 cm represents 20 m s^{-1}

Draw the scale diagram for the vector triangle. Angles **must** be measured accurately.



Worked Example 3 continued



Measuring the length of the resultant gives ~ 9.5 cm. Using the scale in reverse to find the speed gives ~190 m s^{-1}.

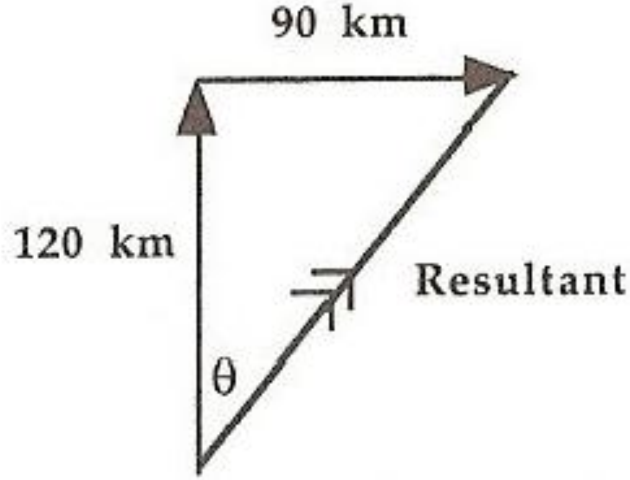
Measuring the angle θ gives ~ 9°.

The velocity of the plane is ~ **190 m s^{-1} at 9° north of east** (or at a bearing of 081°).

Worked Example 4

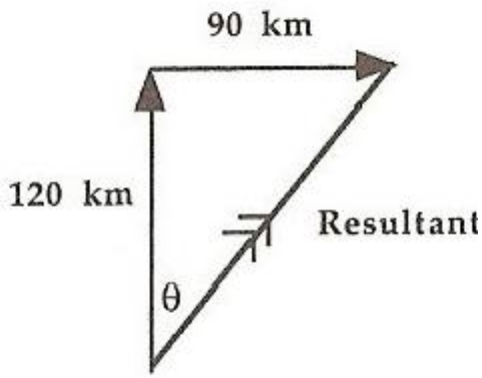
What is the resultant displacement of an aircraft which flies 120 km north and then 90 km east?

Draw a sketch diagram of the situation.



Decide from the sketch whether the problem can best be solved by calculation or if a scale diagram is required.

Worked Example 4 continued



In this case, since the two vector quantities are at right angles, the resultant can be found using Pythagoras' theorem:

$$|\text{Resultant}| = \sqrt{120^2 + 90^2} = \sqrt{22\,500} = 150 \text{ km}$$

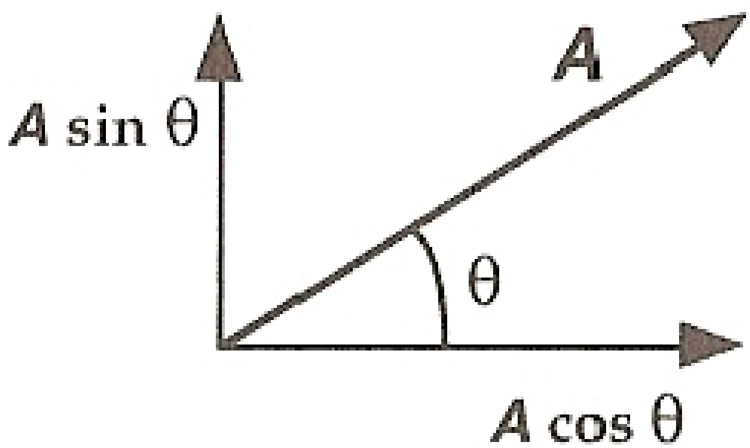
Since displacement is a vector quantity the direction must be calculated as well:

$$\tan\theta = \frac{90}{120} \quad \Rightarrow \quad \theta = 36.9^o$$

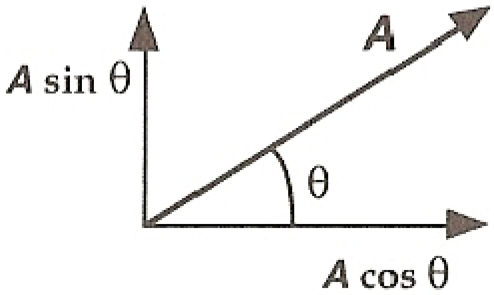
*The resultant displacement is **150 km at 36.9º east of north** (or at a bearing of 036.9º).*

Vector Components

Any vector, ***A***, can be replaced by two other vectors, acting at the same point, which are at right angles to each other. These two vectors, of magnitudes ***A*** cos θ and ***A*** sin θ, are called the **rectangular components** of the original vector:

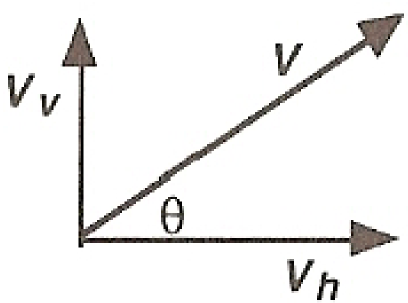


Vector Components continued



The cos component always touches the given angle.

This can be used to find the effect of a vector in a particular direction, e.g. for a projectile, such as a golf ball, sent off at a velocity V at angle θ to the horizontal:



where

$V_v = V \sin \theta$

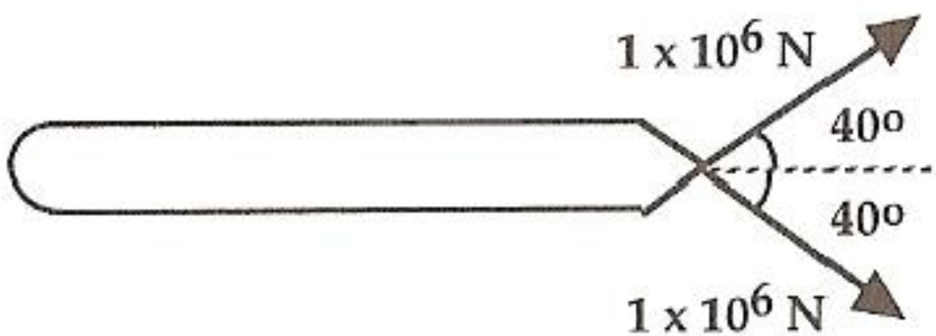
$V_h = V \cos \theta$

The resultant of a number of forces acting on an object is the single force which will produce the same effect on the object.

Worked Example 5

Two tugs are pulling a tanker into dock. The angle between the two tow-lines is 80^{o} and each tug exerts a pull of 1×10^6 N. What is the size and direction of the resultant force exerted by the tugs on the tanker?

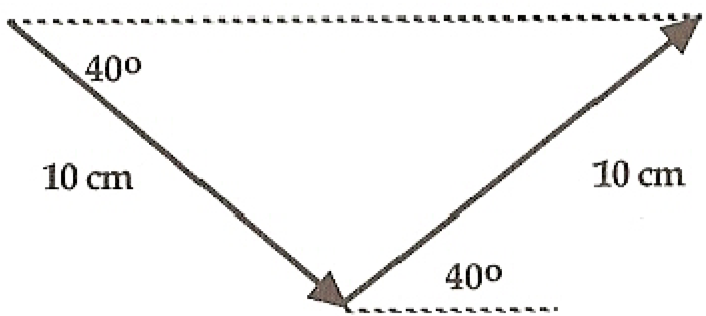
Draw a sketch diagram of the situation.

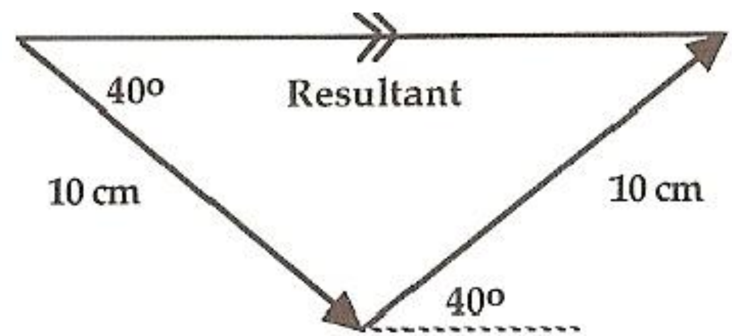


Worked Example 5 continued

Choose a suitable scale for the vector triangle, e.g. 10 cm represents 1×10^6 N. Draw a vector, with an arrow head, to scale, in a suitable direction to represent one force. Then from the head of this vector, draw a second vector in the correct direction to represent the second force.

The resultant is ~**1.53×10^6** N, in a direction midway between the two tugs (at 40° to each tug).



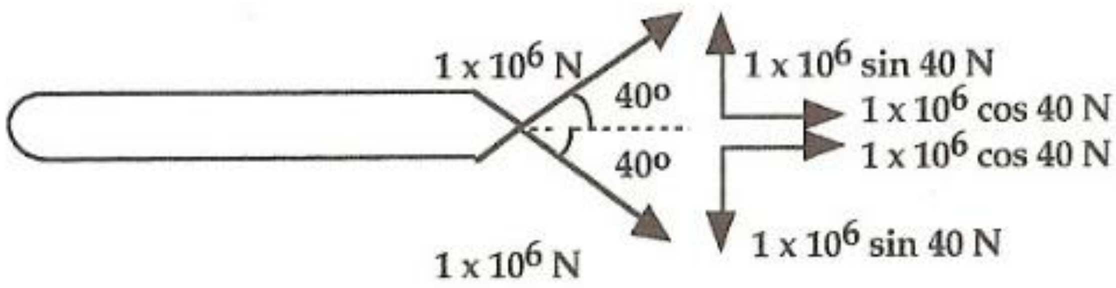


Measuring the length of the resultant gives ~ 15.3 cm.
Using the scale in reverse to find the force gives ~1.53×10^6 N

The resultant is ~**1.53×10^6** N, in a direction midway between the two tugs (at 40° to each tug).

Worked Example 5 by Calculation

Replace the two vectors by their components.



Add the components at right angles to the tanker and in the direction of the tanker's motion.

At right angles, the sin components cancel as they are in opposite directions.

The cos components add together.

Resultant $= 2 \times (1 \times 10^6 \cos 40)$

$= \mathbf{1.53 \times 10^6\ N}$

The direction is midway between the two tugs.