

Uncertainty

Year 11 Physics

When you measure a length with a metre rule where the smallest scale division is 1 mm, you should estimate your measurements to the nearest 0.1 mm (i.e. 'read between the lines' of the rule). The uncertainty should then show the lower and upper limits of confidence in your estimate. A reading for an experienced user might be 22.6 ± 0.05 mm. This would indicate that the measurer was confident that the reading was not as high as 22.7 mm or as low as 22.5 mm.

Consider a typical measurement of 22.6 ± 0.1 mm. This has an **estimated absolute uncertainty** of ± 0.1 mm. **When you add or subtract measurements**, you can estimate the absolute uncertainty of the result by adding the absolute uncertainties of the measurements.

$$22.6 \pm 0.1 \text{ mm} - 14.2 \pm 0.1 \text{ mm} = 8.4 \pm 0.2 \text{ mm}$$

We can express uncertainty as a percentage. This can be useful when you evaluate an investigation, because the measurement having the largest percentage uncertainty tends to contribute most to inaccurate results. In a measurement the uncertainty can be expressed as a percentage by using the relationship:

$$\text{percentage uncertainty} = \frac{\text{absolute uncertainty}}{\text{measurement}} \times 100\%$$

$$\text{percentage uncertainty in } 22.6 \pm 0.1 = \frac{0.1}{22.6} \times 100\% = \pm 0.4\%$$

$$\text{percentage uncertainty in } 14.2 \pm 0.1 = \frac{0.1}{14.2} \times 100\% = \pm 0.7\%$$

In this case, the smaller measurement has a greater percentage uncertainty than the larger measurement. When you multiply or divide measurements, you can estimate the absolute uncertainty of the result by adding the percentage uncertainties of the measurements.

For example:

The area contained in a rectangle 22.6 ± 0.1 mm by 14.2 ± 0.1 mm is $(22.6 \times 14.2) \text{ mm}^2 = 390.92 \text{ mm}^2$.

We have already calculate the percentage uncertainties above, so the uncertainty in the area must be $\pm(0.4 + 0.7)\% = \pm 1.1\%$.

The area is thus $390.92 \text{ mm}^2 \pm 1.1\%$.

1.1% of 390 is 4.3, so the absolute uncertainty in the area is $\pm 4 \text{ mm}^2$.

We should therefore report the area as $391 \pm 4 \text{ mm}^2$.

Significant figures represent an approximate way of indicating how confident you are of your measurements. The final digit carries an element of uncertainty. Thus, the number of significant figures is the number of certain digits, plus the first digit in which there is some uncertainty.

The following guidelines can help you work out the number of significant figures in a measurement or quoted number:

- All non-zero digits are significant.
- Zeros between non-zero digits are significant: e.g. 4003 has 4 s.f.
- Zeros to the right of a decimal point and following a non-zero digit are significant: e.g. 1.1003 has 5 s.f. and 0.01030 has 4 s.f.
- If only zeros occur to the left of a non-zero digit, those zeros are not significant: e.g. 0.002 has 1 s.f.
- Numbers such as 2000, where the decimal point is not included, are ambiguous: e.g. 2000 could have 1, 2, 3 or 4 s.f. In such numbers, the number of significant figures should be agreed beforehand.
- Scientific notation avoids ambiguity: e.g. 31 400 may have 3, 4 or 5 s.f. but 3.14×10^4 has 3 s.f.
- In calculations, the final result should contain no more significant figures than the data with the least number of significant figures.