

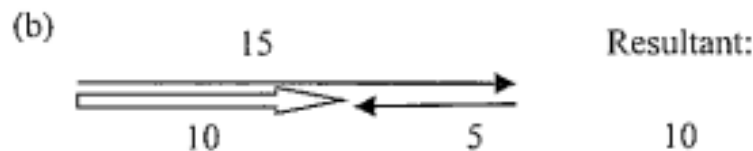
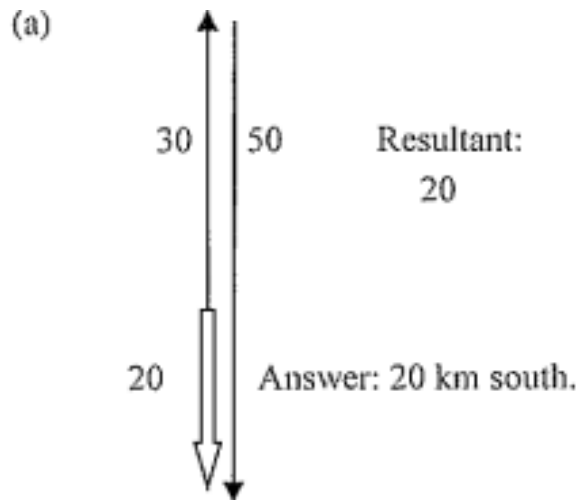
PQ 2 Q and A

Vectors

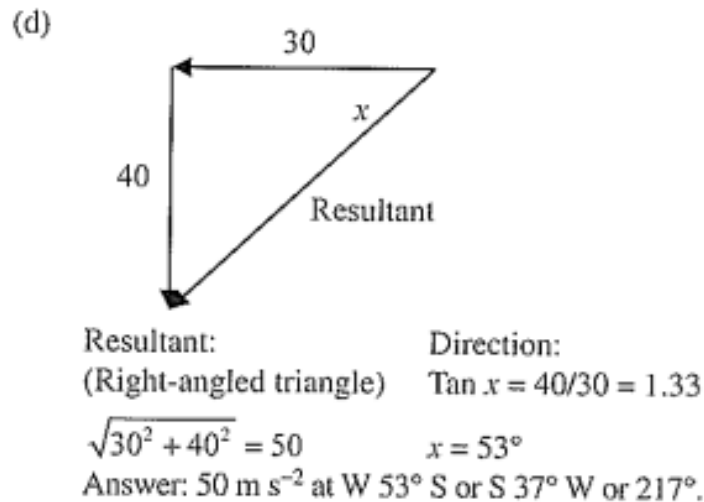
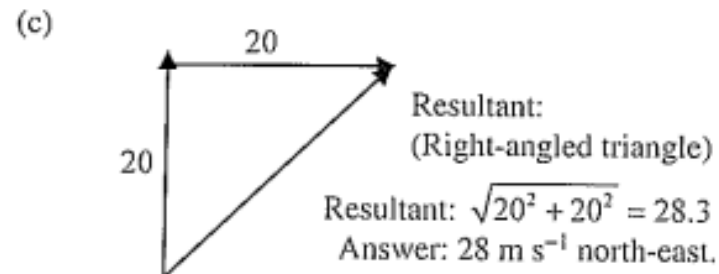
# Q1

Calculate the resultants of the following combinations of vectors. Your answer must include both magnitude and direction.

- (a) 30 km north + 50 km south.
- (b) 15 newtons east + 5 newtons west.
- (c) 20 m s<sup>-1</sup> north + 20 m s<sup>-1</sup> east.
- (d) Acceleration of 30 m s<sup>-2</sup> west and 40 m s<sup>-2</sup> south.

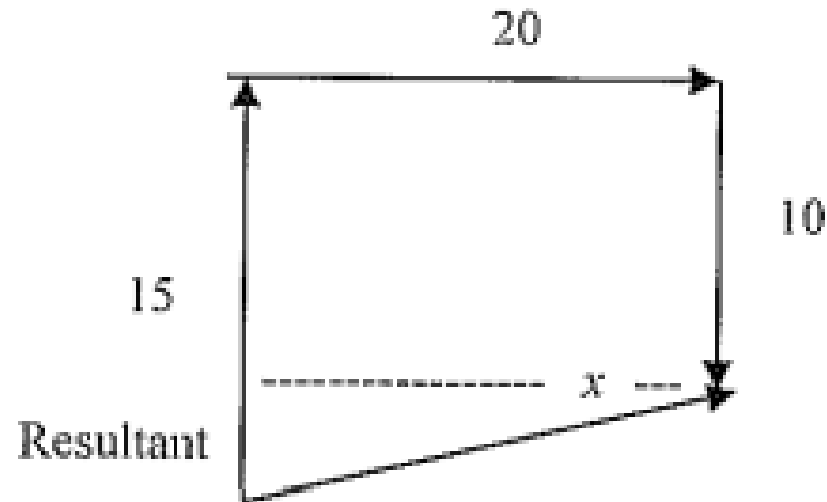


Answer: 10 newtons east.



# Q2

A ship sailed 15.0 km north, then 20.0 km east, followed by 10.0 km south. Calculate its final displacement from its starting point.



Resultant:

$$\sqrt{20^2 + 5^2} = 20.6$$

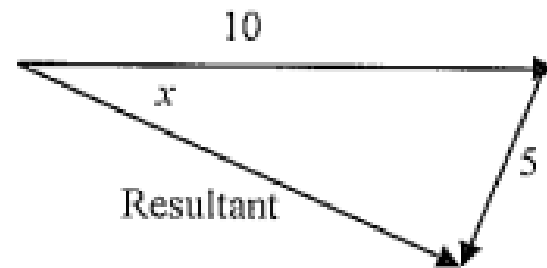
Answer: 21 km at E  $14^\circ$  N or  $76^\circ$ .

Direction:

$$\tan x = 5/20 = 0.25$$

# Q3

A motor boat heads due east at  $10.0 \text{ m s}^{-1}$  on a current moving towards south  $30^\circ$  west at  $5.0 \text{ m s}^{-1}$ . Find the resultant velocity of the boat.



Resultant:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$R^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \times \cos 60^\circ$$

$$R = 8.7 \text{ m s}^{-1}$$

Direction:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{R}{\sin 60^\circ} = \frac{5}{\sin x}$$

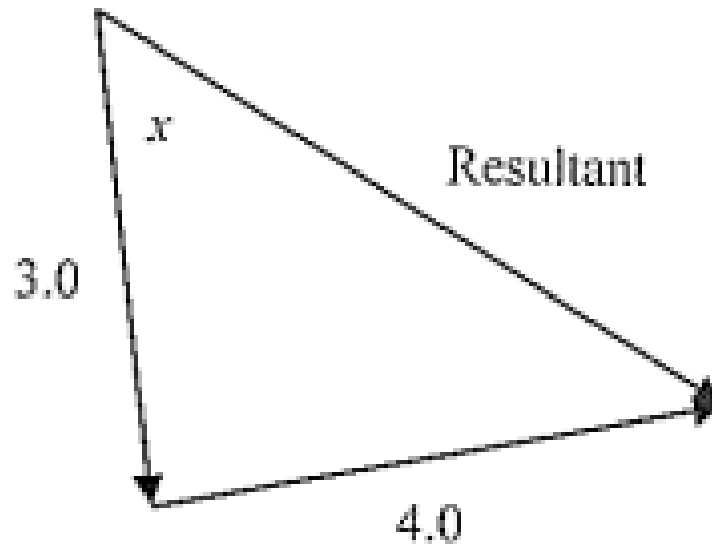
$$\sin x = \frac{5 \sin 60^\circ}{8.7} = 0.50$$

$$x = 30$$

Answer:  $8.7 \text{ m s}^{-1}$  at E  $30^\circ$  S or  $120^\circ$ .

# Q4

Two cars collide on an icy road. Car A had a momentum of  $3.0 \times 10^4 \text{ kg m s}^{-1}$  in a direction south  $20^\circ$  east and car B had a momentum of  $4.0 \times 10^4 \text{ kg m s}^{-1}$  in a direction east  $20^\circ$  north. After the collision the cars locked together and slid off with a momentum equal to the vector sum of the original momenta. Calculate this vector sum.



Resultant:  
(Right-angled triangle)  
 $5.0 \times 10^4$

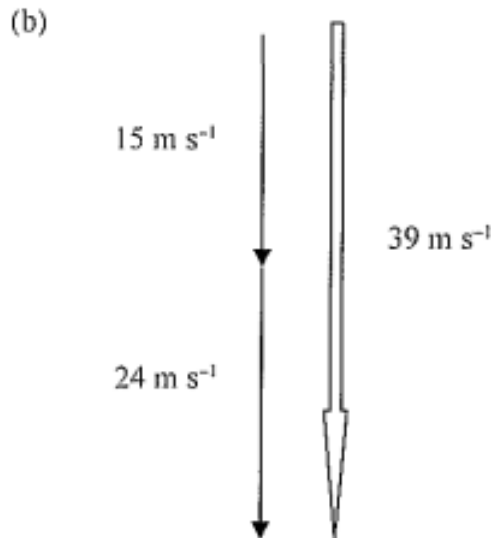
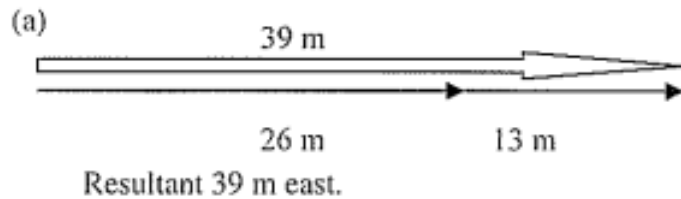
Direction:  
 $\tan x = 4.0/3.0 = 1.33$   
 $x = 53^\circ$

Answer:  $5.0 \times 10^4 \text{ kg m s}^{-1}$  at S  $73^\circ$  E or E  $17^\circ$  S or  $107^\circ$ .

# Q5

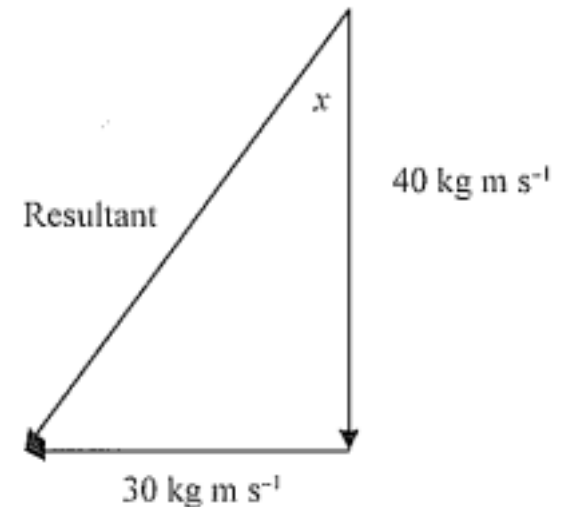
Calculate the resultants of the following. Your answers must contain both magnitude and direction.

- (a) 26 m east – 13 m west.
- (b)  $15 \text{ m s}^{-1}$  south –  $24 \text{ m s}^{-1}$  north.
- (c)  $40.0 \text{ kg m s}^{-1}$  south –  $30.0 \text{ kg m s}^{-1}$  east.
- (d) 20.0 newtons at  $200^\circ$  – 15.0 newtons at  $110^\circ$ .
- (e)  $48 \text{ m s}^{-2}$  west  $30^\circ$  south –  $36 \text{ m s}^{-2}$  north  $40^\circ$  west.



Resultant  $39 \text{ m s}^{-1}$  south.

- (c)  $45.0 \text{ kg m s}^{-1}$  south –  $30.0 \text{ kg m s}^{-1}$  east  
 $= 45.0 \text{ kg m s}^{-1}$  south +  $30.0 \text{ kg m s}^{-1}$  west



Resultant:  
(345 right-angled triangle)

$50 \text{ kg m s}^{-1}$

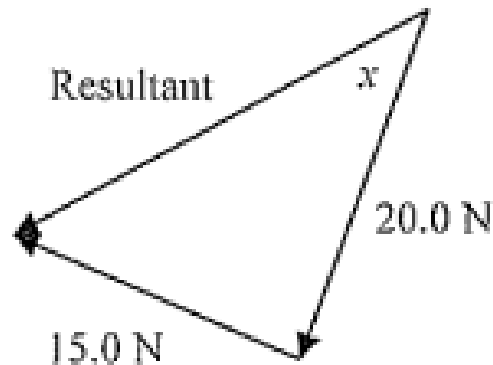
Direction:

$$\tan x = \frac{30}{40} = 0.75$$

$$x = 36.9^\circ$$

# Q5 continued

- (d) 20.0 newtons at  $200^\circ$  – 15.0 newtons at  $110^\circ$   
 $= 20.0$  newtons at  $200^\circ$  + 15.0 newtons at  $290^\circ$ .



Resultant:

20.0 N

(345 right-angled triangle)

25 N

Direction:

$200.0 + 36.9 = 236.9^\circ$

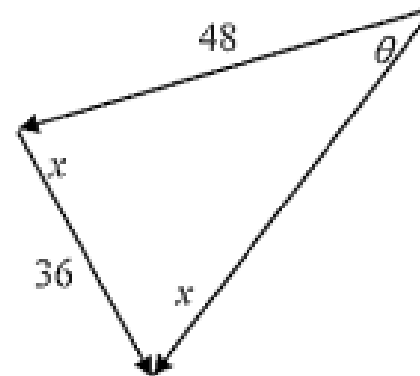
Answer: 25.0 newtons at  $236.9^\circ$ .

$$\tan x = 15.0/20.0$$

$$= 0.75$$

$$x = 36.9^\circ$$

- (e)  $48 \text{ m s}^{-2}$  W  $30^\circ$  S –  $36 \text{ m s}^{-2}$  N  $40^\circ$  W  
 $= 48 \text{ m s}^{-2}$  W  $30^\circ$  S +  $36 \text{ m s}^{-2}$  S  $40^\circ$  E



Resultant:

$x = 80^\circ$

$$\text{Resultant}^2 = 48^2 + 36^2 - 2 \times 48 \times 36 \cos 80^\circ$$

$$= 2999.87$$

$$\text{Resultant} = 55 \text{ m s}^{-2}$$

Direction:

$$\cos \theta = \frac{48^2 + 55^2 - 36^2}{2 \times 48 \times 55}$$

$$= 0.7638$$

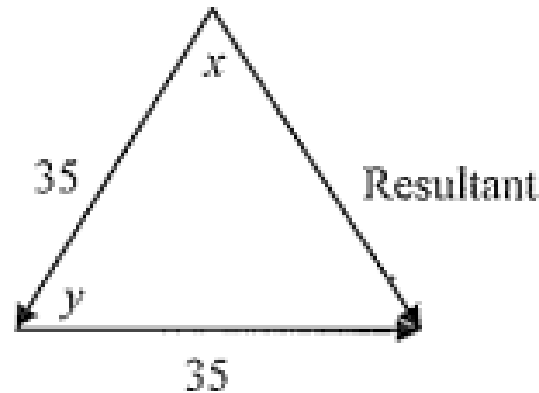
$$\theta = 40.2^\circ$$

Answer is  $55 \text{ m s}^{-2}$  at W  $70.2^\circ$ S or S  $19.8^\circ$ W or  $199.2^\circ$ .

# Q6

Two forces act on an object. One force is 35 newtons towards the west. Calculate the other force given that the resultant of the two forces is 35 newtons towards  $240^\circ$ .

35 newtons at  $240^\circ$  – 35 newtons west  
= 35 newtons at  $240^\circ$  + 35 newtons east



$$x = y = 60^\circ$$

Resultant:

(equilateral triangle)

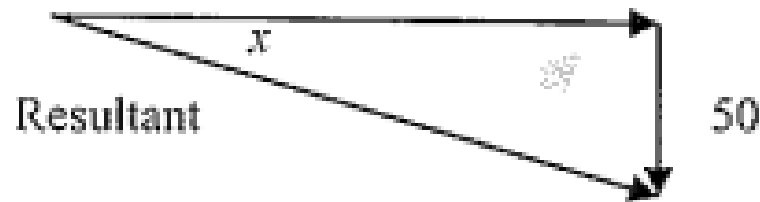
35 newtons

Answer is 35 newtons at  $150^\circ$ .

# Q7

An aeroplane is to fly 200 km to the east in 1 hour. There is, however a wind blowing at 50 km h<sup>-1</sup> from the south. Calculate the direction and speed the aeroplane has to fly to reach its destination on time.

$$\begin{aligned} &200 \text{ km h}^{-1} \text{ east} - 50 \text{ km h}^{-1} \text{ from south} \\ &= 200 \text{ km h}^{-1} \text{ east} + 50 \text{ km h}^{-1} \text{ from north} \end{aligned}$$



Resultant:  
(Right-angled triangle)  
 $\text{Resultant}^2 = 200^2 + 50^2$   
Resultant = 206

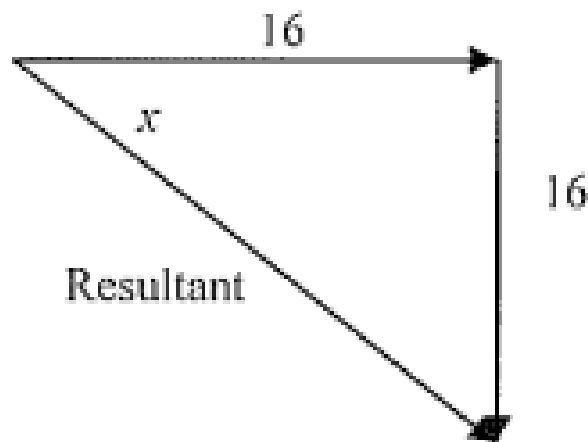
Direction:  
 $\tan x = 50/200 = 0.25$   
 $x = 14^\circ$

Answer is 206 km h<sup>-1</sup> at E 14° S.

# Q8

A car travelling north at  $16 \text{ m s}^{-1}$  turned a corner to the east maintaining the same speed. Calculate the change in velocity.

$$16 \text{ m s}^{-1} \text{ towards east} - 16 \text{ m s}^{-1} \text{ towards north} \\ = 16 \text{ m s}^{-1} \text{ towards east} + 16 \text{ m s}^{-1} \text{ towards south}$$



Resultant:  
(Right -angled triangle)

$$\text{Resultant}^2 = 16^2 + 16^2$$

$$\text{Resultant} = \sqrt{512}$$

$$\text{Resultant} = 22.6$$

Answer is  $22.6 \text{ m s}^{-1}$  at E  $45^\circ$  S.

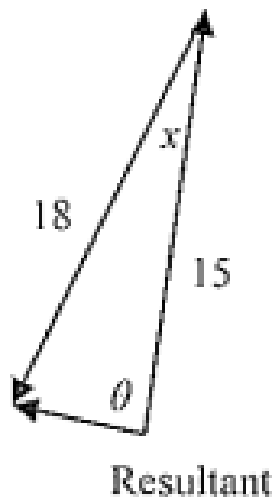
Direction:

$$x = 45^\circ$$

# Q9

In order to keep going in a direction of north  $10^\circ$  east at  $15 \text{ km h}^{-1}$  a boat had to sail on a heading of north  $20^\circ$  east at  $18 \text{ km h}^{-1}$  to counteract the current. Calculate the strength of the current in magnitude and direction.

$$15 \text{ km h}^{-1} \text{ at N } 10^\circ \text{ E} - 18 \text{ km h}^{-1} \text{ at N } 18^\circ \text{ E} \\ = 15 \text{ km h}^{-1} \text{ at N } 10^\circ \text{ E} + 18 \text{ km h}^{-1} \text{ at S } 18^\circ \text{ W}$$



$$x = 8^\circ$$

Resultant:

$$\text{Resultant}^2 = 18^2 + 15^2 - 2 \times 18 \times 15 \times \cos 8^\circ \\ = 103.4$$

$$\text{Resultant} = 10.2$$

Direction:

$$\cos \theta = \frac{15^2 + 10.2^2 - 18^2}{2 \times 15 \times 10.2} \\ = 0.014$$

$$\theta = 89^\circ$$

Answer is  $10.2 \text{ km h}^{-1}$  at N  $79^\circ$  W.