Momentum

Recall That

Momentum is the product of velocity and mass

$$\vec{p} = m\vec{v}$$

"how hard it is to stop a thing"

Momentum is a vector quantity, with the same direction as the velocity

Forces applied over time periods create impulses.

An impulse is equal to the net force on the object times the time period over which this force is applied.

Change in Momentum

$$\Delta p = m \Delta v$$

$$\Delta p = m(v - u)$$

$$\Delta p = mv - mu$$

$$\Delta p = p^f - p^I$$

Line 1: The change in momentum equals the mass times the change in velocity.

Line2: The change in velocity is the final velocity minus the original velocity.

Line 3: Distribute the m.

Line 4: The change in momentum equals the final momentum minus the original momentum.

On the first line we state that the change in momentum is equal to the mass times the change in velocity.

In line two we change delta v to the quantity of the final velocity minus the original velocity, as one can do with any delta quantity.

The mass is distributed over the two velocities on line three. Thus, the right side obviously ends up as the difference between two momenta. The momentum shown as the mass times the final velocity, i.e., mv', is the final momentum, and the momentum shown as the mass times the original velocity, i.e., mv, is the original momentum. So, on the right side of this third line we see the final momentum minus the original momentum.

On the fourth line we simply restate the change in momentum using the symbol p' for the final momentum and p for the original momentum.

Impulse: Where It Comes From?

- Derive impulse from the equation F=ma

$F = ma$	Line 1: Force equals mass times acceleration.
$F = m\frac{\Delta v}{\Delta t}$	Line2: Definition of acceleration.
$F\Delta t = m\Delta v$	Line 3: Algebraic rearrangement, the force multiplied by the time period equals the mass multiplied by the change in velocity.

- 1st line is our familiar equation F=ma
- 2nd line expresses the ***acceleration*** as the ***change in velocity divided by the change in time***. Ie basic definition of acceleration.
- 3rd line is arrived at through algebra by multiplying each side of the equation by delta t, canceling it on the right, effectively moving it over to the left.

$$F\Delta t = m\Delta v$$

The Impulse — The Change in Momentum

- ***The left side of the third line is called the impulse on the object.***
- ie impulse is equal to the net force times the length of time over which that force is applied.
- ***The right side of the third line is called the change in momentum.***
- So, we say the impulse equals the change in momentum.

Impulse unit		Momentum unit
Newton-second	=	kilogram-meter per second

Change in momentum

A change in momentum will occur when an external force is applied

$$
\begin{aligned}
\Delta p &= p^f - p^I \\
&= mv - mu \\
&= m(v-u) \\
&= m\Delta v
\end{aligned}
$$

Worked Example

A 1600 kg truck is moving forward at 60.0 kmh^{-1} when it enters a school zone and needs to drop to 40.0 kmh^{-1}.

a) What is the initial momentum of the truck?

b) What is the change in momentum of the truck?

$u = 60\ \text{kmh}^{-1} = \frac{60 \times 1000}{} = 16.67\ \text{ms}^{-1}$

b) $v = 40\ \text{kmh}^{-1} = \frac{40 \times 1000}{60 \times 60} = 11.11\ \text{ms}^{-1}$

$p = mu$

$= 1600 \times 16.67 = 2.67 \times 10^{4}\ \text{kgms}^{-1}$ forward

b) $v = 40\ \text{kmh}^{-1} = \frac{40 \times 1000}{60 \times 60} = 11.11\ \text{ms}^{-1}$

$\Delta p = m \Delta v$

$= 1600\,(11.11 - 16.67) = -8.90 \times 10^{3}\ \text{kgm}$

$\therefore \Delta p = 8.90 \times 10^{3}\ \text{kgms}^{-1}$ backwards.

Equal Momentum

Consider two trucks moving at the same speed. Truck one applies the brakes to stop while truck two hits a barricade.

Both trucks experience the **same change in momentum** but experience different forces and times over which this change in momentum occurs.

$\Delta p = m(v-u)$ $\quad a = \frac{v-u}{t} \quad \therefore \; v-u = at$

$\Delta p = \underline{m\,a}\,t \quad ; \quad F = \underline{ma}$

$\Delta p = \underline{Ft}$

$I = \Delta p$

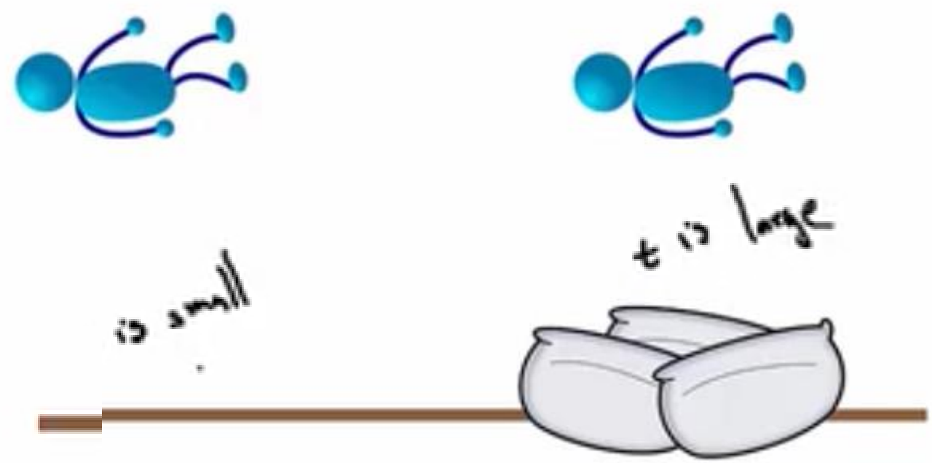
Relationship between I, F and t

$Ns \equiv kg\,ms^{-1}$

$$I = Ft$$

(N)(s)

F↓ t↑ → less damage



For any given impulse (change in momentum), force and time are inversely proportional