MOMENTUM AND IMPULSE

The momentum of an object is given by:

Momentum = mass × velocity of the object.

Momentum = mv

(Momentum: $kg\ m\ s^{-1}$; m: kg; v: $m\ s^{-1}$)

Note: momentum is a **vector** quantity.
The **direction** of the momentum is the same as that of the velocity.

Conservation of Momentum

When two objects collide it can be shown that momentum is conserved **provided** there are no external forces applied to the system. **LEARN**!

For any collision:

Total momentum of all objects before = total momentum of all objects after.

Elastic and inelastic collisions

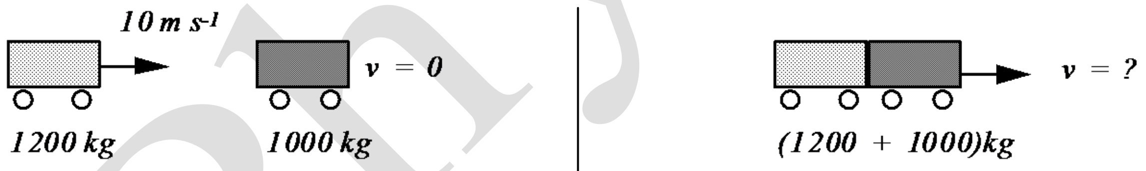
An elastic collision is one in which **both** kinetic energy and momentum are conserved.
An inelastic collision is one in which **only** momentum is conserved. **LEARN**!

Example

a) *A car of mass 1200 kg travelling at 10 $m\ s^{-1}$ collides with a stationary car of mass 1000 kg. If the cars lock together find their combined speed.*

b) By comparing the kinetic energy before and after the collision, find out if the collision is elastic or inelastic.

Draw a simple sketch of the cars before and after the collision.



BEFORE — AFTER

a)

Before: *Momentum* $= mv = 1200 \times 10 = 12000\ kg\ m\ s^{-1}$

After: *Momentum* $= mv = (1200 + 1000)\,v = 2200v\ kg\ m\ s^{-1}$

Total momentum before = Total momentum after

$$12000 = 2200v$$

$$\frac{12000}{2200} = v$$

$$v = 5.5\ m\ s^{-1}$$

b)

Before: $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 1200 \times 10^2 = 60{,}000\ J$

After: $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 2200 \times 5.5^2 = 33{,}275\ J$

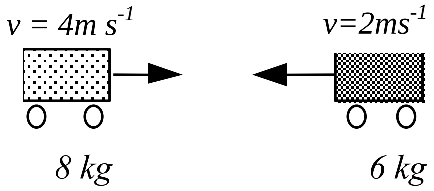
Kinetic energy is not the same, so the collision is ***inelastic***

Vector nature of momentum

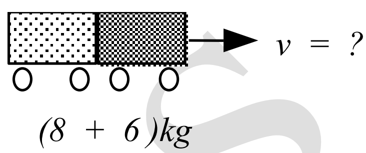
Remember momentum is a vector quantity, so direction is important. Since the collisions dealt with will act along the same line, then the directions can be simplified by giving:

momentum to the right a positive sign and
momentum to the left a negative sign.

Example
Find the unknown velocity below.



BEFORE
Momentum = mv
$= (8 \times 4) - (6 \times 2)$
$= 20$ kg m s^{-1}



AFTER
Momentum = mv
$= (8 + 6)v$
$= 14v$

Total momentum before = Total momentum after

$$20 = 14v$$

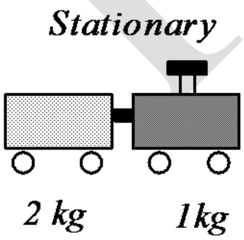
$$v = \frac{20}{14} = 1.43 \text{ m s}^{-1}$$

Trolleys will move to the right at 1.43 m s^{-1} since v is positive.

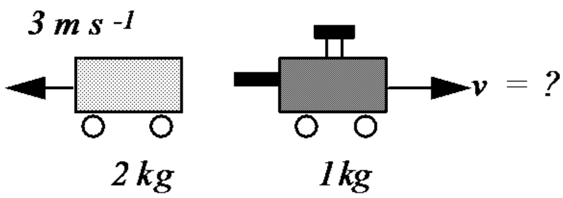
Explosions

A single stationary object may explode into two parts. The total initial momentum will be zero. Hence the total final momentum will be zero. Notice that the kinetic energy increases in such a process.

Example
Two trolleys shown below are exploded apart. Find the unknown velocity.



BEFORE
Total momentum = mv
= 0



AFTER
Total momentum = mv
$= -(2 \times 3) + 1v$
$= -6 + v$

Total momentum before = Total momentum after

$$0 = -6 + v$$

$v = 6$ m s^{-1} to the right (since v is positive).

Impulse

An object is accelerated by a force F for a time, t. The unbalanced force is given by:

$$F = ma = \frac{m(v - u)}{t} = \frac{mv - mu}{t}$$

$$\text{Unbalanced force} = \frac{\text{change in momentum}}{\text{time}} = \text{rate of change of momentum}$$

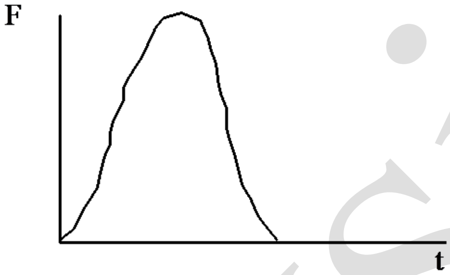
$$\boxed{Ft = mv - mu}$$

Pay attention to signs!

The term Ft is called the **impulse** and is equal to the change in momentum.

Note: the unit of impulse, Ns will be equivalent to kg m s^{-1}.

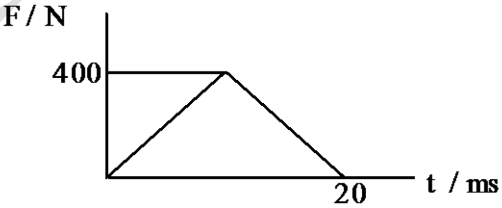
The concept of **impulse** is useful in situations where the force is not constant and acts for a very short period of time. One example of this is when a golf ball is hit by a club. During contact the unbalanced force between the club and the ball varies with time as shown below.



Since F is not constant the impulse (Ft) is equal to the **area** under the graph. In any calculation involving impulse the unbalanced force calculated is always the <u>average</u> force and the maximum force experienced would be greater than the calculated average value.

Examples

1. *In a snooker game, the cue ball, of mass 0.2 kg, is accelerated from the rest to a velocity of 2 m s^{-1} by a force from the cue which lasts 50 ms. What size of force is exerted by the cue?*
   u = 0 v = 2 m s^{-1} t = 50 ms = 0.05 s   m = 0.2 kg   F = ?
   Ft = mv - mu
   F × 0.05 = 0.2 × 2   F = 8 N
2. *A tennis ball of mass 100 g, initially at rest, is hit by a racket. The racket is in contact with the ball for 20 ms and the force of contact varies over this period as shown in the graph. Determine the speed of the ball as it leaves the racket.*



Impulse = Area under graph

$$= \frac{1}{2} \times 20 \times 10^{-3} \times 400 = 4 \text{ N s}$$

2 1

u = 0 m = 100 g = 0.1 kg v = ?

Ft = mv - mu = 0.1v

4 = 0.1 v

v = 40 m s^{-1}

3. *A tennis ball of mass 0.1 kg travelling horizontally at 10 m s^{-1} is struck in the opposite direction by a tennis racket. The tennis ball rebounds horizontally at 15 m s^{-1} and is in contact with the racket for 50 ms. Calculate the force exerted on the ball by the racket.*

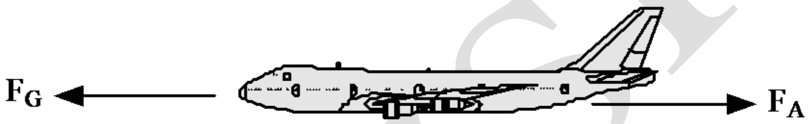
$m = 0.1$ kg $u = 10$ m s^{-1} $v = -15$ m s^{-1} *(opposite direction to u)*
$t = 50$ ms $= 0.05$ s
$Ft = mv - mu$
$0.05 F = 0.1 \times (-15) - 0.1 \times 10$
$= -1.5 - 1 = -2.5$

$F = \dfrac{-2.5}{0.05} = -50$ N *(Negative indicates force in opposite direction to initial velocity)*

Newton's 3rd Law and Momentum

Newton's 3rd law states that if an object A exerts a force (ACTION) on object B then object B will exert an equal and opposite force (REACTION) on object A.
This law can be proved using the conservation of momentum.
Consider a jet engine expelling gases in an aircraft.

Let F_A be the force on the aircraft by the gases
and F_G be the force on the gases by the engine (aircraft).



Let the positive direction be to the **left** (direction of Fa)

In a small time, let m_G be the mass of the gas expelled and
m_A be the mass of the aircraft.

total momentum before = total momentum after
$O = m_G v_G + m_A v_A$
$m_G v_G = - m_A v_A$ [v_B and v_A in opposite directions]
$(m_G v_G - O) = -(m_A v_A - O)$
Change in momentum of gas = - (change in momentum of aircraft)

Changes in momentum of each object are equal in size but opposite in direction.

If forces act in time, t

$$\text{Force} = \frac{\text{change in momentum}}{t}$$

$$F_G = \frac{(m_G v_G - O)}{t} \qquad FA = \frac{(m_A v_A - O)}{t} \qquad (u_A = u_G = O)$$

But $(m_G v_G - O) = (m_A v_A - O)$ from above

$F_G = - F_A$ since t is the same for the engine and gas.

The forces acting are equal in size and opposite in direction.