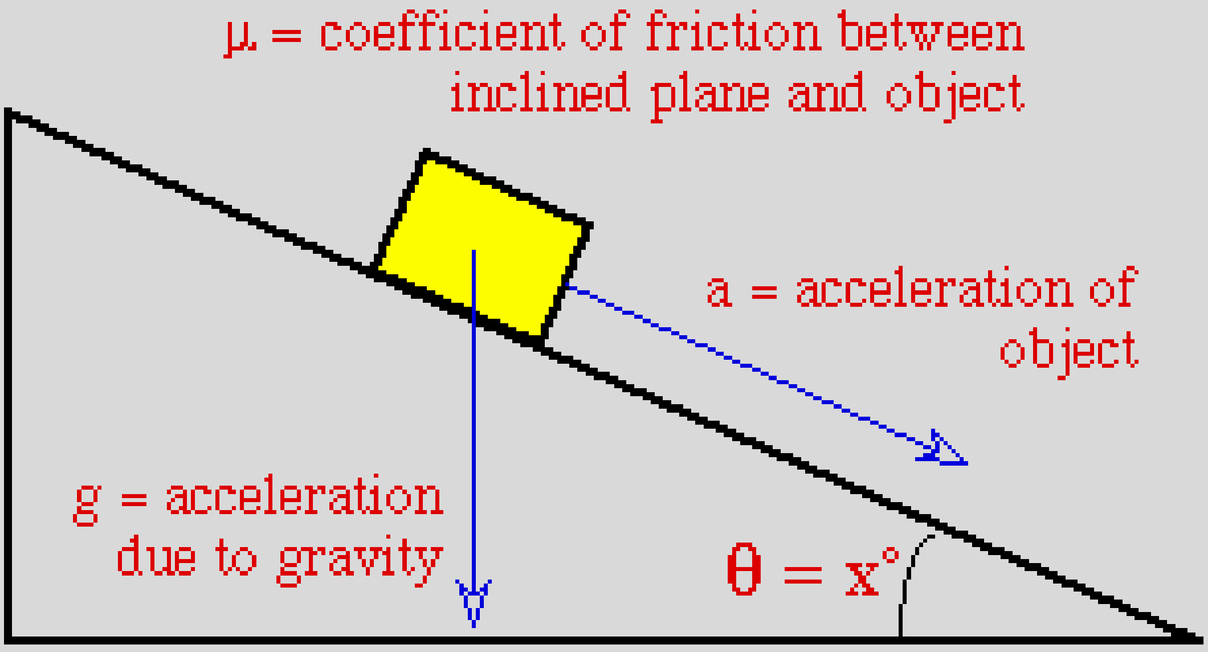
Inclined Planes



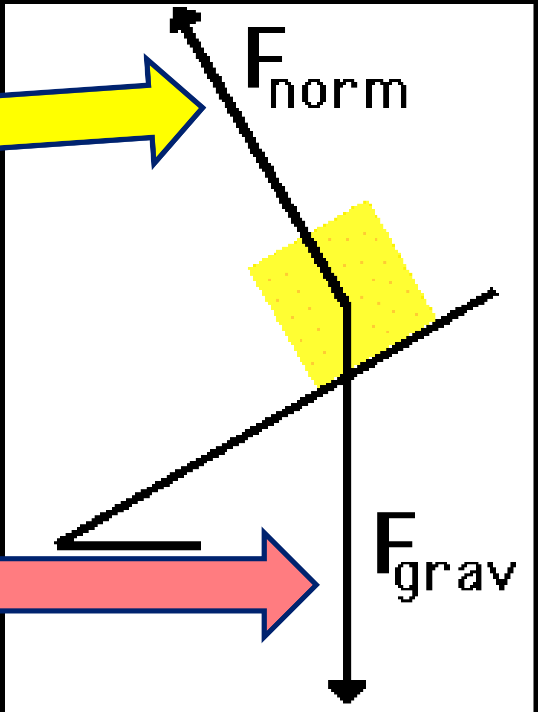
Acceleration of an object on an inclined plane

Background Information

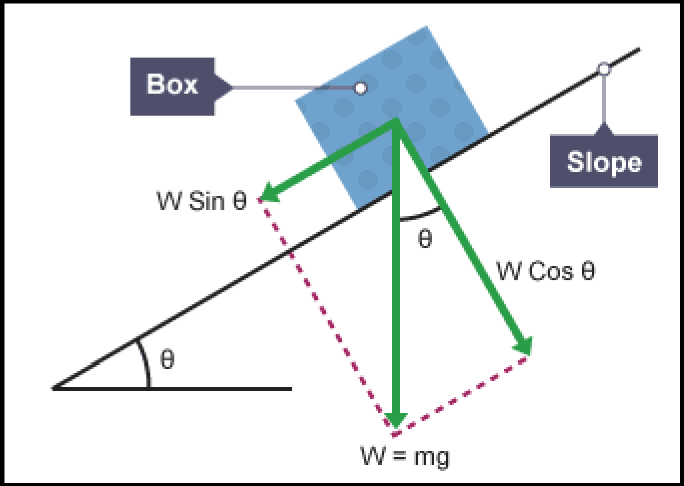
- The greater the angle of the inclined surface, the faster an object will slide down the incline
- There are always at least 2 forces acting on an object on an inclined plane F_{grav} and F_{norm}

The normal force is always perpendicular to the inclined surface

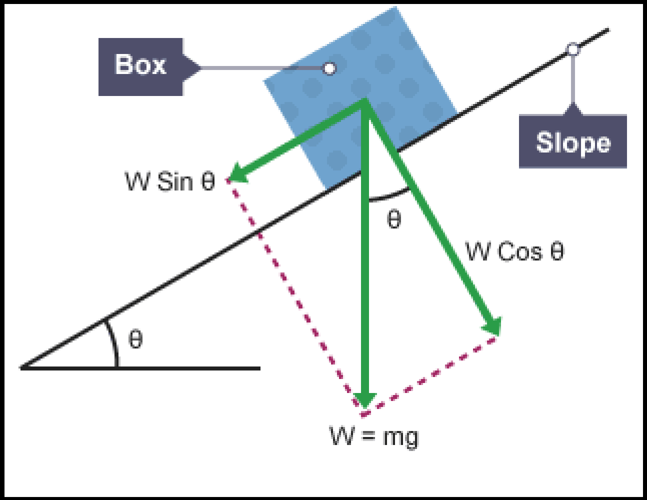
The gravitational force (WEIGHT) is always in downward direction



Vector components for an object on a slope



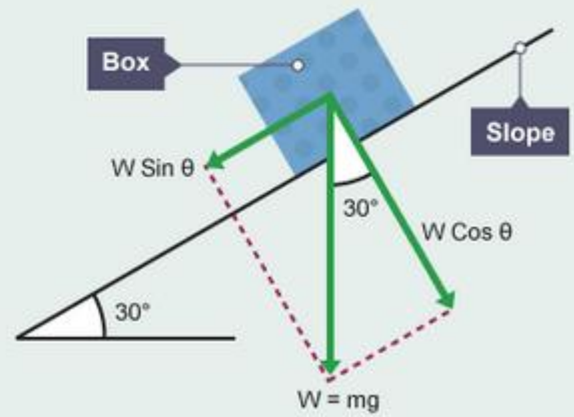
- In this case the weight is the single force and it is resolved into two independent components,
- One acting along the slope F_s
- Other acting perpendicular to the slope F_p.



- Expressed mathematically:
- Component of weight parallel to slope equals: $W \sin\theta = mg \sin\theta$
- Component of weight perpendicular to slope equals: $W \cos\theta = mg \cos\theta$

Worked Example

- A 10 kg box slides down a frictionless slope. The slope is at 30° to the horizontal.
- Find the component of the weight acting parallel to the slope.



The component of weight parallel to the slope is:

$W \sin\theta = mg \sin\theta$

$= 10 \times 9.8 \times \sin 30^{\circ}$

$= 49N$

Worked example continued

- Now find the acceleration of the box down the incline.

$$F = 49N$$

$$m = 10kg$$

$$a = ?$$

$$F = ma$$

$$49 = 10 \times a$$

$$a = 4.9ms^{-2}$$

Worked example continued

- If the slope is steeper, how does this affect the two components?

$mg\,sin\,\theta$ parallel to slope will increase and $mg\,cos\,\theta$ will decrease

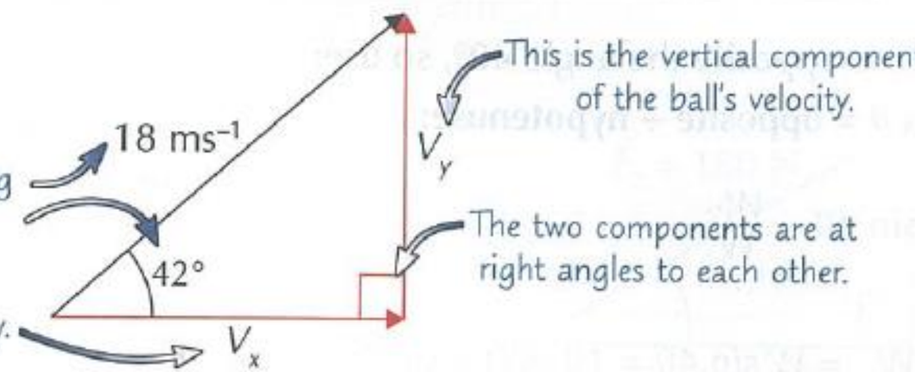
- How will this affect the acceleration?

$mg\,sin\,\theta$ will cause increased force and $mg\,cos\,\theta$ will decrease and friction as there will be less force pushing the block onto the surface.

Worked Example 2

A footballer kicks a ball. The initial velocity of the ball is 18 ms^{-1} at an angle of 42° above the horizontal.
Calculate the vertical and horizontal components of the ball's initial velocity.

Draw a diagram to show what's going on.
This will help you to see what you need to find.



The horizontal component of the vector is **adjacent** to the angle, so you'd use **cos θ = adjacent ÷ hypotenuse.**

$$\cos\theta = \frac{V_x}{V} \quad \text{so} \quad \boxed{V_x = V\cos\theta}$$

The vertical component of the vector is **opposite** the angle, so you'd use **sin θ = opposite ÷ hypotenuse.**

$$\sin\theta = \frac{V_y}{V} \quad \text{so} \quad \boxed{V_y = V\sin\theta}$$

Worked Example 2

Use the formulas to find the horizontal and vertical components.

$$V_x = V\cos\theta = 18 \times \cos 42 = 13.376...$$

$$V_y = V\sin\theta = 18 \times \sin 42 = 12.044...$$

So the horizontal component of the ball's initial velocity is 13 ms^{-1} (to 2 s.f.) and the vertical component of the ball's initial velocity is 12 ms^{-1} (to 2 s.f.) upwards.